Towards an Information-Neutral Voting Scheme That Does Not Leave Too Much To Chance

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Abstract

We examine a new voting procedure that allows voters to express their preferences in the form of a voting strategy — a first-order function which specifies a vote. We examine the calculations necessary to formulate such voting strategies, and present examples which illustrate the effects of risk. The voting framework we discuss allows rational voters to vote strategically using decision-theoretic techniques to select their optimal strategies, even if they have no prior information about the preferences of others and are unfamiliar with decision theory; moreover, randomness can be introduced into the framework to increase resistance to manipulability while maintaining outcomes that do not leave too much to chance.

Introduction

Voting theorists have long been aware that it is not always in the best interest of voters to vote for their most sincerely preferred candidates. Voting schemes in which voters can obtain a more preferred outcome by voting *strategically* rather than *sincerely* are dubbed *manipulable*. And it has been well established that most voting schemes used in elections with three or more alternatives (including all reasonably "attractive" voting schemes previously investigated) are manipulable [1, 11, 12, 21]. On the other hand, there are well-known systems that rely on randomness to elicit sincere voting behavior [1]; however, the arbitrariness present in such systems renders them generally unacceptable to voting populations.

Studies have shown empirical evidence that voting systems are manipulated by voters in real elections. Brams and Merrill [5] analyzed data collected in the 1992 National Election Study and estimated that 34.5 percent of Perot supporters did not vote sincerely in the 1992 Presidential election because they felt they could make better use of their votes by voting for their second-choice candidates. Cain [6] developed a model of strategic voting in the British electorate that suggests strategic manipulation does occur, especially when voters want to avoid "wasting" their votes or when the race is only close between two of the parties in a three party election. And Black [3] analyzed data from the 1968 and 1972 Canadian Federal elections and estimated that 12 percent of the voters in these elections voted for their second-choice party. It is difficult to determine how many of the voters participating in the analyzed elections voted sincerely even though they could have benefited from voting strategically. And it is uncertain whether these sincere voters voted sincerely out of ignorance of the preferences of others, ignorance of how to use preference information to their best advantage, a desire to show support for an underdog candidate, or a belief that insincere voting is dishonest.

In this paper we will examine some of the problems that may arise when voters use information about the preferences of other voters to manipulate an election. We will then sketch an *information-neutral* voting system that would minimize the possibilities for such manipulation while relying minimally on chance. This system allows voters to optimize their votes without knowledge of the preferences of others. It also improves analysts' abilities to interpret election results accurately.

Background and Motivation

Early criticism of manipulable voting systems equated manipulation with dishonesty. The eighteenth century mathematician Jean-Charles de Borda is said to have responded to criticism that his Borda count voting scheme was manipulable by saying, "My scheme is only intended for honest men" [2]. But later theoreticians dismissed Borda's assumption that people are honor-bound to vote for their most preferred candidate. In an 1876 pamphlet[1] written well before the development of game theory, C. L. Dodgson suggested that an election be thought of "more as a game of skill than a real test of the wishes of the electors." He proposed that "as my own opinion is that it is better for elections to be decided according to the wish of the majority than of those who happen to have most skill in the game, I think it desirable that all should know the rule by which this game may be won." Although Dodgson elaborated on the rules for winning certain types of games, little additional work was done in this area until 1953 when Farquharson set out to apply game theory to voting procedures [9].

Farquharson began by introducing the concept of a *voting strategy.* Farquharson described a voting strategy as a plan made prior to an election that prescribes the course of action a voter should take given any contingency that can arise. In a single round election, a strategy is simply the voter's plan for how to cast a single ballot. However, in a multiple round election such as one that involves choosing between two alternatives at a time, a strategy must include a plan for all pairs of alternatives that could possibly be presented. Farquharson also discussed *sincere* voting, in which voters always vote for their most preferred alternatives, and *sophisticated* voting in which voters select utility maximizing strategies that take into account the strategies of the other voters [9].

In this paper we follow Farquharson's terminology and distinguish sophisticated voting from strategic voting, using the following definitions:

Sincere vote: A vote such that for any two alternatives under consideration, the voter has given greater or equal weight to the more preferred of those alternatives.

Strategic vote: A vote that is rational, but not sincere. Often such a vote will be determined through decision-theoretic analysis, which takes into account the preferences of the other voters.

Sophisticated vote: A vote determined optimal through game theoretic analysis, which takes into account the likely strategies of the other voters. Note that all sophisticated votes are also strategic. Sophisticated votes may be viewed as higher-order functions than simple strategic votes.

We will also refer to strategic or sophisticated votes as *insincere votes*.

The application of game theory to voting theory brought on a variety of new approaches to studying voting schemes and voter behavior. It led to further investigation of the conditions under which voting schemes

[1] Dodgson's unpublished pamphlet "A method of taking votes on more than two issues," is reproduced as an appendix to [2].

might be immune to manipulation, and it led to the development of models designed to provide rational choice explanations for voter behavior.

Several voting theorists, including Farquharson and Dummett [8], conjectured that it would be impossible to find a voting scheme immune to manipulation. Indeed, in the early 1970s, Gibbard and Satterthwaite independently proved that all non-dictatorial voting schemes with at least three possible outcomes are manipulable [12, 21]. Gibbard also demonstrated by example the existence of a "mixed decision scheme" that is neither manipulable nor dictatorial, and can allow more than two outcomes. In this scheme each voter submits a ballot containing a vote for a single alternative. One ballot is then selected at random, and the alternative specified in that ballot is declared the winner. This scheme is unsuitable for most purposes because it relies heavily on chance. Gibbard suggested that further work be done to explore decision schemes that do not leave "too much to chance." This suggestion was taken up by Barbera [1], who showed that "the only selection methods that can be both nondictatorial and nonmanipulable are those in which chance plays an extensive role." Gardenfors [11] also extended the work of Gibbard and Satterthwaite, focusing on voting schemes that do not necessarily select a single outcome. He showed that most of these schemes are manipulable, and those which are not tend to be very indecisive.

While these theorists were exploring the extent to which voting schemes are manipulable, other voting theorists sought to determine the optimal strategies for voters to use. McKelvey and Ordeshook developed "A General Theory of the Calculus of Voting" in which they derived decision rules that could be used by voters to determine their optimal strategies [15]. This article extended "A Theory of the Calculus of Voting" in which Riker and Ordeshook derived decision rules that could be used to determine whether it was rational for a particular voter to vote at all [20]. The theory was further generalized by Hoffman in "A Model For Strategic Voting" [13].

All of the decision models cited here are expected-utility models that require voters to consider their personal preferences as well as the probable preferences of the rest of the electorate. Information about the preferences of others allows voters to determine the relative probability of each candidate winning the election. When voters are able to determine such probabilities, they are said to be making decisions under *risk*. Without information about the preferences of others, voters are not able to determine the probabilities of the various contingencies. When this occurs voters are said to be making decisions under *uncertainty*. Merrill showed that the sincere strategy is always the optimal strategy when decisions are made under uncertainty in plurality, Borda, and approval voting elections [17]. This is likely the case for other voting schemes as well. Merrill's results suggest that voters are only able to manipulate voting systems when they are making decisions under risk. When voters have no information about the preferences of others and therefore must make their decisions under uncertainty, they cannot manipulate the voting system.

Others have also pointed out that the fact that a voting system is manipulable does not imply that it will actually be manipulated. As Gibbard explained:

> …to call a voting scheme manipulable is not to say that, given the actual circumstances, someone really is in a position to manipulate it. It is merely to say that, given some possible circumstances, someone could manipulate it [12].

The circumstances required for manipulation are:

1. a set of preferences for each voter such that at least one voter or group of voters can obtain a more preferred outcome by voting insincerely,

2. at least one voter who has sufficient information about the preferences of other voters to determine that he or she can obtain a more preferred outcome by voting insincerely, and

3. at least one voter who has sufficient knowledge about how to solve utility maximization problems to determine his or her utility maximizing strategy.

In an election where the required circumstances for manipulation are present, not all voters who could benefit from voting strategically will have sufficient knowledge and information to formulate a utility maximizing strategy. As we have discussed, the ability to manipulate is dependent on the individual voter's ability to gather information about the preferences of the other voters — but there may be costs associated with gathering this information [3]. Depending on the type of election, money, education, access to media, time, and political alliances may be necessary resources for gaining accurate voter preference information. Because of the difficulty some voters may have in obtaining this information themselves, it is conceivable that voters might accept inaccurate estimates — perhaps propagated by certain candidates or their supporters — as the truth. Indeed, it has been reported that supporters of Pat Buchanan actively sought out poll takers during the 1992 and 1996 Republican Presidential primaries so that pre-election polls would show more support for Buchanan than actually existed [7]. In addition, as noted by Black [3], some voters "may hold probability estimates that are inflated or clearly erroneous or the product of the simple mimicking of the views of a close friend or spouse." Moreover, the ways that poll data and other preference information flow through society and are interpreted by voters are not well understood [4].

Thus a significant problem with manipulable voting systems is that they allow voters with more accurate information about the preferences of others to use their votes more effectively than other voters, essentially granting these voters a weighted vote. This is contrary to the principles espoused by many democratic countries and organizations in which all people are given the same amount of say in the electoral process, regardless of their means. Those who subscribe to these principles often expect that by granting each individual one vote, they are granting all people equal voting power. However, this is not necessarily the case when some voters have the means to obtain information about the preferences of other voters which is unavailable to everyone. These voters may use this information to formulate optimal voting strategies which cannot be identified in the absence of such information.

Another problem with manipulable voting systems is that not all voters understand the formulation of utility maximizing functions. Thus even if all voters have equal information about the preferences of others, those who understand the formulation procedure have more power than those who do not. Riker [19] notes that when strategic voting occurs frequently, the meaning of social choices "may consist simply of the tastes of some people (whether majority or not) who are skillful or lucky manipulators."

Finally, manipulable voting systems frustrate attempts at analyzing and interpreting election results because analysts cannot distinguish between voters who voted sincerely and those who voted insincerely. Thus election results give little indication as to whether a winning alternative had strong support from those who voted for that alternative. As noted by Knight and Johnson [14], "Social choice theorists demonstrate that any electoral outcome is at least partly an artifact of the aggregation mechanism through which it is produced. Therefore, electoral results always require interpretation and justification." Insincere voters add a degree of uncertainty to such interpretation.

In the next section we will develop a framework for a voting system in which all voters are given equal access to information about the preferences of others, voters may vote strategically without having to understand the formulation of utility maximizing strategies, and election results reveal the sincere preferences of the electorate in addition to the election outcome.

A Framework for Declared-Strategy Voting

If there were a way to elicit sincere preference information from all voters prior to an election, this information could be made public and all voters would have the opportunity to use it in formulating their strategies. In the absence of sincere preference information, information about the strategies the other voters plan to use (sincere or otherwise) is still useful. However once the information is published, voters will adjust their strategies to take into account the preferences of others. Myerson and Weber [18] describe a series of polls in which voters are given the results after each poll. Eventually, they predict, a *voting equilibrium* may arise in which "the perceptions arising from the publication of the poll lead the voters to behave in a manner that in turn justifies the predictions of the poll." They then go on to prove that at least one voting equilibrium must exist in all elections. In this scenario, all voters have equal access to the information that they need to vote strategically. However, conducting an election in this manner is unattractive because it would require voters to go repeatedly to the polls. This sort of election is likely to take a long time and have low voter turnout.

One way to side-step the problem of supplying voters with equal access to information is to allow them to cast votes that are contingent on the votes of others. While some parliamentary voting rules allow absent voters to cast proxies that are contingent on the votes of others (for example, the system used by the French National Assembly [9]), contingency voting is not a feature of most voting schemes. Contingency voting has a fundamental problem in that non-contingent votes must be counted before contingent votes; if all voters wish to cast contingent votes there is no way to count any votes without introducing an element of chance to decide which votes to count first. Another problem with contingency voting is the difficulty in evaluating complex contingency rules, especially when the number of voters is large or when votes are tallied at many geographically distributed precincts. But contingency voting is nonetheless appealing because it allows voters without voter preference information to vote strategically. In addition, examination of contingency votes may provide insight into the true preferences of the electorate that is not otherwise available when voters vote insincerely.

The advent of high speed computers and computer networks makes it feasible to tally contingency ballots in a reasonable amount of time. In addition, computers can handle complicated vote aggregation methods, making possible voting procedures not previously considered. With that in mind, we now sketch the design of an information-neutral voting system that employs contingency voting. We shall call this system a *declared-strategy* system because voters declare the computations used to determine their vote. The computation represents the voter's strategy in formulating a vote, rather than simply the outcome of that decision process. Following game-theoretic practice, we assume that each voter decides on a *strategy* prior to the election that includes a specification of how the vote will be cast given any contingency. A declared strategy is a first-order function of the "state" of the election. In our system, voters will vote by submitting their strategies to the election computer. The computer will evaluate the voters' strategies using the current election tally and other state information. The strategies will then be aggregated to determine the election outcome.

In analyzing the declared-strategy voting system for rational votes, the following components must be defined:

- the set of permissible strategies that voters may submit,
- the method for calculating the probabilities of the various contingencies, and
- the method for aggregating the strategies of the entire electorate.

Permissible Strategies

Before determining what strategies should be permitted and how they should be formulated, we considered the strategies that rational voters might like to use. We assume that voters are rational if they take the actions they believe are most likely to lead to their most preferred outcome. We are not concerned with whether it is rational for them to prefer that outcome. As already discussed, several authors have suggested that voters may use expected-utility models involving their personal preferences and the probable preferences of the rest of the electorate to determine their rationally optimal strategies. In fact, Black [3] used the approach outlined by McKelvey and Ordeshook [15] to model the 1968 and 1972 Canadian Federal elections and found "significant empirical support for the role of probabilities in the multicandidate calculus of voting."

McKelvey and Ordeshook [15] deduced that in N-candidate elections one can setup N difference equations that calculate the expected gain in utility associated with a given voter voting for a particular candidate rather than abstaining. Assuming that the voter has established a set of utility values $(u_1, u_2, \ldots u_N)$ specifying his or her personal utility for candidate N winning the election, and assuming that the probability of a three-way tie is negligible, the difference equations for a plurality election which selects a single winner can be written in the form:

$$E_k - E_0 = \sum_{i \neq k} p_{ik}(u_k - u_i)$$

where E_k is the expected utility associated with voting for candidate k, E_0 is the expected utility associated with abstaining, and p_{ik} (which we shall refer to as a *pivot probability*) is the probability that the voter will be *decisive* in creating or breaking a tie between i and k. Note that the voter affects the election result if and only if he or she casts a vote that creates or breaks a first-place tie. A rational voter will vote for the candidate k that maximizes the $E_k - E_0$ equations.

Consider, for example, a three candidate plurality election. The voter can combine the difference equations for his or her two most preferred candidates (denoted C_1 and C_2) to form the equation:

$$E_1 - E_2 = 2p_{12}(u_1 - u_2) + p_{13}(u_1 - u_3) - p_{23}(u_2 - u_3)$$

Thus $E_1 - E_2$ can be used to determine the candidate for which it is rationally optimal to vote:

$$E_1 - E_2 > 0\text{: vote for } C_1$$
$$E_1 - E_2 < 0\text{: vote for } C_2$$

The McKelvey and Ordeshook model is ambiguous about how the voter should vote if $E_1 - E_2 = 0$. When this occurs, the voter cannot expect to be better off voting for C_1 rather than C_2 or vice versa. For example, this situation occurs when the voter is indifferent between all candidates. In this case the voter might decide to abstain, make a random selection, adopt a bandwagon strategy (voting for the candidate most likely to win), or adopt an underdog strategy (voting for the candidate most likely to lose). Although unlikely, this situation can also occur when the voter is not indifferent between all candidates. In this case the voter may choose any of the above options or vote for C_1 as a show of support.

Hoffman [13] paraphrased the $E_1 - E_2$ equation into a *critical value* formula:

$$A^* = \frac{u_1(2p_{12} + p_{13}) + u_3(p_{23} - p_{13})}{2p_{12} + p_{23}}$$

indicating that a voter should vote for C_1 if $u_2 < A^*$ and C_2 if $u_2 > A^*$. We shall use Hoffman's formulation throughout the remainder of this paper.

In developing our declared-strategy voting system, we assume that rational voters will develop declared strategies consistent with the expected-utility model. Therefore, to accommodate these voters, our system need only accept strategies based on the expected-utility equations for the voting procedure being used (simple plurality, approval, etc.) and the number of candidates under consideration. In fact, a tie-breaking rule (for example, bandwagon strategy) plus a single strategy function in terms of utilities and pivot probability estimates should be sufficient for representing the declared strategy of any rational voter. This approach even accommodates those voters who wish to vote despite being indifferent between all candidates. These voters may submit equal utility values for all candidates and have their votes based solely on their tie-breaking rules. One situation where this might prove useful involves voters who are indifferent between the candidates running in their party's primary but want to vote for whichever candidate is most likely to win in order boost the winner's share of the vote and make their party appear more unified.

The expected-utility calculus can also be extended for elections that use decision rules other than plurality and that select more than one winner. For example, Merrill [16] presented a model in which a voter's utility differences for a pair of candidates was multiplied by the difference in votes that the voter awarded to each member of that pair of candidates. This accommodates a voting system such as Borda count in which the voter casts $N-1$ votes for his or her most preferred candidate, $N-2$ votes for his or her second most preferred candidate, and so on.

For the purposes of this paper, all voters in a given election are essentially using the same (rational) voting strategy with differing parameters; thus, voters need only submit the relevant parameters as part of their ballots. In this system we will ask voters to submit their utility values for each candidate on a scale of 0 to 10, as well as their tie-breaking rules. Depending on the probability calculation method used, we may also require the voters to submit a number that indicates their attitude toward risk. The other parameters—the probability values—can be calculated on a voter's behalf as explained in the next section. Thus, voters need not understand the process of formulating an expected-utility maximization function to use this system.

Note that we ask voters to submit their sincere utility values for all candidates. Our system will calculate the optimal decision-theoretic strategy based on these values, enabling voters to vote strategically while simultaneously revealing their sincere preferences. Thus it is generally advantageous for voters to submit their sincere utility values. However, as we will discuss later, voters who have the necessary knowledge to formulate sophisticated strategies might not find it to their advantage to reveal their sincere utility values (and will act accordingly).

Pivot Probability Calculations

While the expected-utility model includes a straightforward decision rule, it does not include a method for calculating pivot probabilities. Indeed this calculation appears to be anything but straightforward. Several approaches to this calculation are presented and analyzed here.

Before discussing these approaches, it is useful to examine the definition of pivot probability in more detail. As already mentioned, a voter's pivot probability for any two candidates is the probability that he or she will be decisive in making or breaking a first-place tie between those two candidates. This probability is approximately the sum of the probabilities associated with each of the possible election outcomes that involve a first-place tie between those two candidates.

The election outcome space can be represented visually as a barycentric coordinate system — an equilateral triangle on the three-dimensional plane $v_1 + v_2 + v_3 = 1$, where v_1, v_2, and v_3 represent the percentage of votes for candidates 1, 2, and 3 respectively. Each of the triangle corners represents one candidate. The closer an outcome point is to a particular corner, the more votes the alternative represented by that corner received. An outcome point in a corner of the triangle represents a "shut-out" in which one alternative received all the votes, while an outcome point in the geometrical center of the triangle represents a three-way tie. As shown in Figure 1, the line segments that bisect the triangle represent two-way ties.

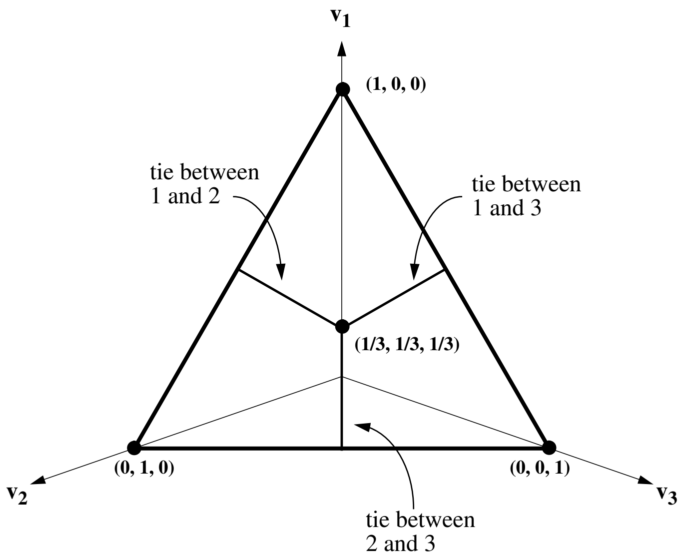

Figure 1: A Three-Dimensional View of the Barycentric Coordinate System

Because the expected-utility function depends on the ratio of the pivot probabilities rather than the actual probabilities themselves, we do not have to find a method for calculating the actual probability of reaching each point in the outcome space; rather it is sufficient to find a method of calculating relative probabilities. Thus the methods we examine here do not necessarily result in the probabilities over the entire outcome space summing to 1. This model can be extended arbitrarily; for example, when representing four-candidate elections the barycentric triangle becomes a solid tetrahedron.

One approach to calculating pivot probabilities involves making a prediction about the likely outcome of the election, plotting that outcome as a point on a barycentric coordinate system, and computing the relative distances between that outcome point and the outcome lines for each of the two-way ties. Black used this method in a model of a three-candidate plurality election in which each voter was assumed to be able to make a reasonable prediction about the election outcome based on the results of previous elections, opinion polls, or other data [3]. In our declared-strategy voting system we can determine the voters' sincere strategies from their utility information, and aggregate these strategies to determine a predicted outcome.

Black assumes that the pivot probability for any pair of candidates is proportional to 1 minus the Euclidean distance between the line segment $\overline{CD}$ representing a first-place tie between those candidates and the predicted outcome point A, as shown in Figure 2. (Note that the example assumes that the predicted outcome ranks candidate 1 in first place, 2 in second place, and 3 in third place. If this predicted ranking does not match the voter's preference ranking, the appropriate substitutions must be made. For example, if the voter had the preference ranking 3 over 2 over 1, the voter's p_{12} would refer to the probability of a tie between candidates 2 and 3.) For p_{12} this distance $|AB|$ is equivalent to the length of the perpendicular line segment $\overline{AB}$ from the predicted outcome point to the first-place tie line $\overline{CD}$. For p_{23} the relevant distance is $|AD|$, the length of the line segment $\overline{AD}$ from the predicted outcome point to the three-way tie point D. For p_{13}

the relevant distance calculation depends on the percentage of votes received by the various candidates in the predicted outcome. If the second place candidate is predicted to receive more votes than the average of the other two candidates (a dominant second place finish) the p_{13} distance is $|AE|$. Otherwise the p_{13} distance is calculated in the same manner as the p_{23} distance. The distance calculation for $|AB|$ can be expressed as:

$$|AB| = [(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2]^{1/2}$$

where $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$. $|AD|$ and $|AE|$ can be calculated in a similar manner.

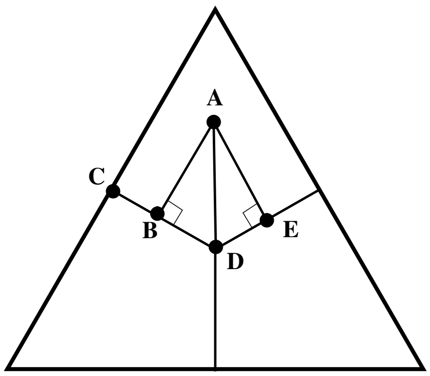

Figure 2: Distances used to calculate Black's probability estimates

One variation on Black's method involves calculating the distance d between the predicted outcome and a given point on a two way tie line. Using this technique, the pivot probability for any two candidates can be found by summing the differences $1 - d$ for every point on the two-way tie line for those candidates. This method makes more intuitive sense than Black's method if one considers what the pivot probabilities really represent.

One problem with both Black's method and the above variation is that they assume that the probability of a tie gets linearly larger the farther away a predicted outcome point is from a two-way tie line. This uniform probability distribution, while simple, does not seem consistent with empirical evidence. For example, these methods appear never to select the strategy of voting for a second-choice candidate unless the second-choice candidate has a utility rating at least 80 percent as high as the first-choice candidate. In addition, these methods do not take into consideration the certainty of the predicted outcome point.

Hoffman [13] offers another approach which allows for the modeling of voting schemes other than simple plurality, and assumes a Gaussian distribution rather than a uniform distribution. As shown in Figure 3, Hoffman specifies the region A_{ij} as the portion of the outcome triangle in which candidate i loses to candidate j by one vote (or less in systems that allow fractional votes). He defines the pivot probability as "the probability that the election result (v_1, v_2, v_3) lies in the region A_{ij}." Thus p_{ij} can be expressed as:

$$p_{ij} = \iint_{A_{ij}} K e^{-D^2/2S^2}$$

where D is the distance from the predicted outcome point to an outcome point in A_{ij}, S^2 is a measure of the uncertainty of the prediction, and K is a constant factor. Because A_{ij} is only one vote wide, p_{ij} can be approximated by using Simpson's rule or another numerical integration technique to integrate over its face.

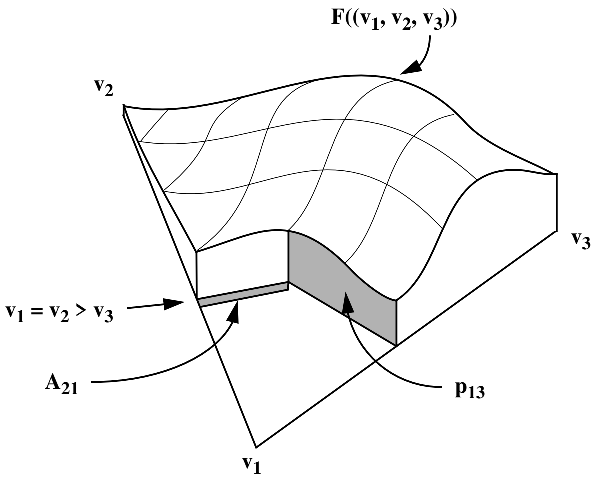

Figure 3: Hoffman's geometry for a 3-candidate election

Hoffman [13] observed that his model might break down if it were used by a large number of voters in a given election without consideration of the dynamic interaction between voters. This is a particular problem for elections in which more than one winner is selected. Hoffman suggests this problem can be overcome by introducing a probabilistic factor into the expected-utility calculation.

Hoffman's approach is appealing because it does not assume a uniform probability distribution and because it takes into account the uncertainty associated with the predicted outcome. Indeed this uncertainty proves to be a significant factor in calculating a voter's optimal strategy. Figure 4 shows the critical value contours we have calculated for S^2 values of .02 and .05. Note that the y-axis shows a normalized utility rating for C_2 such that $100 = u_1 \geq u_2 \geq u_3 = 0$. The figure illustrates that the more certain the prediction, the more willing voters should be to vote for C_2 rather than C_1. For example, given a predicted outcome P of $(.15, .4, .45)$ the optimal strategy when $S^2 = .05$ is to vote for C^2 if $u_2 \geq 59$. However under the same circumstances but with $S^2 = .02$, it is optimal to vote for C_2 if $u_2 \geq 24$.

In our declared-strategy voting system we have several options for obtaining S^2 values. Normally such values are based on the size of a sample in relation to the size of an entire population. But because our predicted outcome point is based on polling the entire population, our prediction derives no uncertainty from sampling error. Rather, the uncertainty is based on not knowing how many voters will find that their optimal strategies are not their sincere strategies.

The simplest way to deal with uncertainty would be to select an arbitrary value, say .026. This particular value has the property that as long as a C_1 is predicted to receive at least 5 percent of the vote, voters will not vote for C_2 unless u_2 is at least 10 percent as large as u_1 (on a 10 point scale, a voter with $u_1 = 10$ must have $u_2 \geq 1$ in order to vote for C_2). Another approach would be to develop a formula for calculating uncertainty based on some aggregation of the utilities submitted by the voters—perhaps taking into account the percentage of voters for whom it might never be optimal to voter for C_2. Such a calculation is also likely to be somewhat arbitrary, however. Still another approach would be to assign random values to S^2 within a reasonable range (say .008 - .2). Finally, voters could select their own uncertainty values based on their attitudes toward risk, formulated perhaps as a function of the "maturity" of the election: number of rounds, position in the voting sequence, etc.

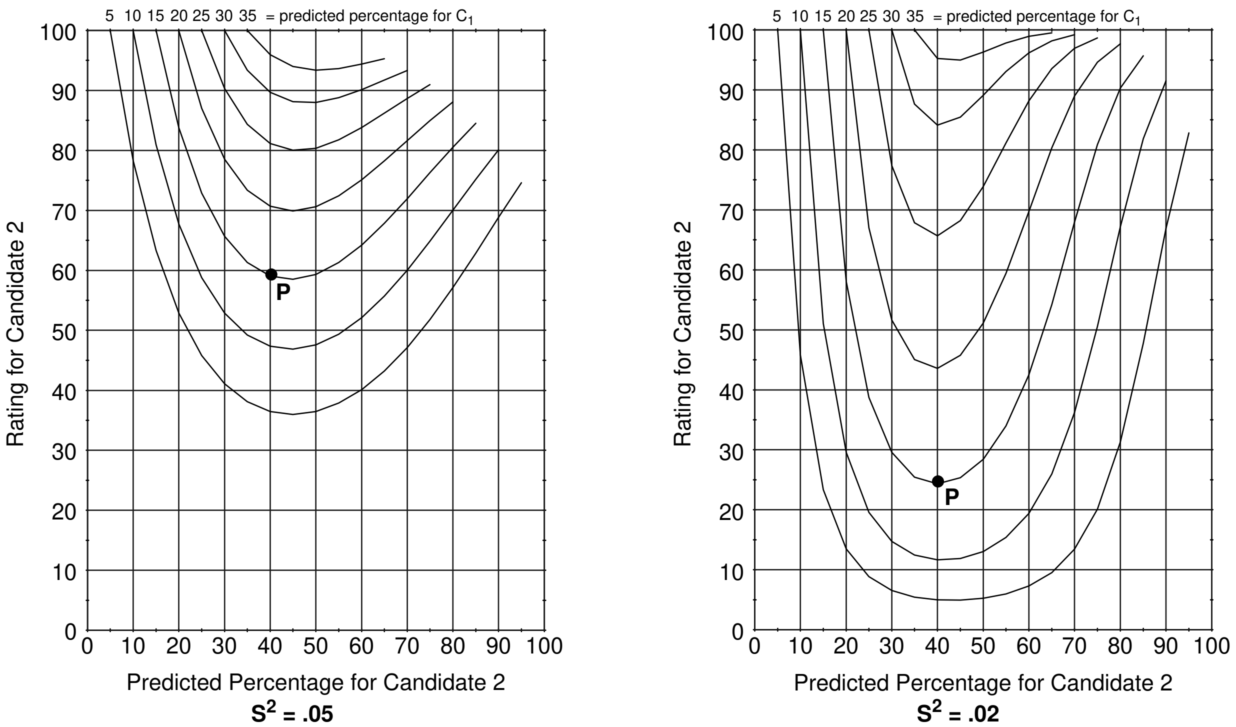

Figure 4: Critical values for 3-candidate plurality election

Strategy Aggregation

The last part of our declared-strategy voting system that must be considered is a method for aggregating the strategies of all voters. We have considered two types of aggregation techniques: ballot-by-ballot and batch. Both of these techniques may be combined with any traditional aggregation technique (plurality, approval, Borda, etc.).

The ballot-by-ballot technique involves making an outcome prediction, evaluating one randomly selected ballot based on that prediction, updating the prediction, and repeating until all ballots have been evaluated. The initial prediction may be based on the sincere outcome point. (Alternately, the initial prediction may be set to 0, however, this seems to result in outcomes which are quite dependent on the first few ballots selected for evaluation.) As the ballots are evaluated, they should be added to a running tally. This technique results in outcomes which are somewhat dependent on the random evaluation order. Thus the election results may not be stable, a problem which render this solution unacceptable for many elections situations.[2]

The batch technique involves making an outcome prediction based on the sincere outcome point, evaluating all ballots based on this prediction, updating the prediction based on the previous evaluations, and repeating until an equilibrium is reached. This technique simulates the situation described by Myerson and Weber [18] in which voters are repeatedly polled until the results of a poll justify the predictions of the previous poll. The batch technique is preferable to the ballot-by-ballot technique because it does not introduce chance into the declared-strategy system.

[2]However, the resistance to manipulability induced by randomness could render this solution acceptable in some settings.

Assessing the Role of Uncertainty in Declared-Strategy Voting

Having sketched the framework for a declared-strategy voting system, we will now describe three ways of assembling the components, each of which results in a system with different properties. The main difference between these systems lies in the way we account for uncertainty. We will explain each system and show the outcome each would produce given an example set of voter preference information.

In our example election, we will assume for simplicity that no voters are indifferent between any pair of candidates; thus we need not consider tie-breaking rules. The utility vectors for our voters are as follows:

Voter Type	Utility Vector	Percentage of Voters
A	(10, 7, 0)	.08
B	(10, 9, 0)	.08
C	(10, 0, 7)	.16
D	(10, 0, 9)	.16
E	(0, 10, 7)	.12
F	(0, 10, 9)	.12
G	(7, 0 ,10)	.14
H	(9, 0 ,10)	.14

Our first system relies on random methods for calculation uncertainty. In this system voters submit their preference information and tie-breaking strategies without knowing the uncertainty factor that will be used. Just prior to the evaluation phase, a random uncertainty factor is calculated. The ballots are then evaluated using the batch technique and the random uncertainty value. This procedure results in a system that elicits sincere preference information from most voters. The only voters for whom it is not necessarily rationally optimal to submit sincere preference information are those who have found a probabilistic sophisticated strategy (this would require prior knowledge of the preferences of the other voters). Because such a strategy is not likely to be found by many voters, this system is highly resistant to manipulation. However, it is also unstable, as the results may vary depending on the random uncertainty value chosen.

For example, with a random uncertainty factor of .027, the evaluation would proceed as follows:

1. Calculate an initial predicted outcome point by evaluating the A^* equation for each voter using equal pivot probabilities. This will result in a predicted outcome point which assumes that all voters will vote sincerely. In this case that point occurs at (.48, .24, .28).

2. Use the predicted outcome point and the uncertainty factor of .027 to calculate first-round pivot probabilities according to Hoffman's method. In this case the pivot probabilities are found to be: $p_{12} = .063$, $p_{13} = .084$, and $p_{23} = .023$.

3. Evaluate the A^* equation for each voter using the new pivot probabilities and calculate a new predicted outcome point. In this example, the F and H voters will select insincere strategies while all the other voters will select sincere strategies. The new predicted outcome point will occur at (.62, .12, .26).

4. Use the first-round predicted outcome point and the uncertainty factor of .027 to calculate second-round pivot probabilities.

5. Evaluate the A^* equations for each voter again, this time using the second-round pivot probabilities. In this round, the E voters join the F and H voters in selecting insincere strategies. The second-round predicted outcome point is (.62, 0, .38).

6. Use the second-round predicted outcome point and the uncertainty factor of .027 to calculate third-round pivot probabilities.

7. Evaluate the A^* equations for each voter again, this time using the third-round pivot probabilities. In this round the H voters return to their sincere strategies, resulting in a third-round predicted outcome point of (.48, 0, .52).

8. Repeating the process for subsequent rounds reveals that the third-round predicted outcome point is an equilibrium outcome.

Depending on the random uncertainty factor selected in this example, equilibrium outcomes may occur at (.48, 0, .52), (.72, .24, 0), or (.62, .24, .14). Voters wishing to use a sophisticated strategy may do so by selecting insincere utilities which will result in a possible outcome set more favorable to their preferred candidates. But unless they can find a sophisticated strategy that will produce the same outcome regardless of the uncertainty factor used, the sophisticated strategy will not always result in the desired outcome. For example, if all voters submit sincere strategies here, candidate 1 will win whenever $S^2 \geq .027$. A and B voters might select a sophisticated strategy in which they would submit the utility vector (9, 10, 0). This would result in a victory for 1 if $S^2 < .031$. Thus, depending on the range of possible random values, voters may be able to increase their chances of achieving a preferred outcome by voting insincerely, but they can rarely guarantee a preferred outcome this way. In addition, by voting insincerely, voters lose their opportunity to make their true preferences known.

Our second system is similar to the first; however, it involves repeating the evaluation process many times, using different uncertainty values for each repetition. The uncertainty values would be selected so as to find all possible outcomes for the set of voter preferences submitted. This system is also highly resistant to manipulation. However, it suffers from being indecisive; this system selects a set of outcomes rather than a single outcome.

Returning to our example, if all voters submit sincere utility vectors, the outcome set will contain the possible outcomes listed above: (.48, 0, .52), (.72, .24, 0), and (.62, .24, .14). This outcome set selects either candidate 1 or candidate 3 as the winner. Voters who submit insincere utility vectors may be able to shift the outcome points or increase or decrease the number of points in the outcome set. In this case, if A and B voters submitted the utility vector (9, 10, 0), the outcome space would contain the points: (.60, .40, 0) (.32, .40, .28). This outcome set selects either candidate 1 or candidate 2 as the winner. Thus supporters of candidate 1 are still not able to remove outcome points which select other candidates from the outcome set. Depending on what procedures are used to further narrow the outcome set, they may have advantaged or disadvantaged candidate 1 by voting insincerely.

Our final system uses a known arbitrary uncertainty value (or alternately an uncertainty value based on voter preferences — this would be computable by anyone who is able to obtain voter preference information prior to the election). This system is both stable and decisive. The only voters for whom it is not necessarily rationally optimal to submit sincere preference information are those who have found a sophisticated strategy. Thus the system resists manipulation (although not as well as the other two systems).

Once again, our example shows that each of three possible outcomes is possible depending on the uncertainty value used. Because voters know the uncertainty value in advance, they may formulate sophisticated strategies in which S^2 is a constant. (Even if we select S^2 based on some aggregation of voter preference information, voters with prior information about the presences of others could deduce this number in advance. Likewise if voters submitted individual risk factors to be used for S^2, some voters may be able to obtain this information in advance; however, such an approach is likely to further complicate the process of formulating a sophisticated strategy and thus reduce the number of voters who do so successfully.) So, if S^2 was selected to be .01, candidate 3 would win if all voters vote sincerely. A and B voters might be tempted to use a sophisticated strategy and submit the insincere utility vector (9, 10, 0). This would result in a victory for candidate 1. However, anticipating that A and B voters will use a sophisticated strategy, E and F voters would also use a sophisticated strategy and submit the insincere utility vector (0, 9, 10). This would result in a victory for candidate 3. In this example, voters cannot change the winner by using a sophisticated strategy (assuming all voters who could benefit from using a sophisticated strategy act accordingly); however, this is not always the case.

Conclusions

The framework for a declared-strategy voting system has been developed and three variations of this system have been described. All three systems allow voters to vote strategically using decision-theoretic techniques to select their optimal strategies, even if they have no prior information about the preferences of others and are unfamiliar with decision theory. However, in order for voters to select sophisticated strategies under these systems, they must have prior information about the preferences of others and be familiar with game theory. The unstable and indecisive systems we described are less susceptible to such sophisticated manipulation than the other system. However, instability and indecisiveness make these systems generally undesirable; thus our third system seems most attractive.

Despite being somewhat manipulable, our system does provide voters with equal access to the information necessary to formulate decision-theoretic strategies. Thus it has a degree of information neutrality. In voting situations where voters are likely to cast strategic votes that are not sophisticated, introducing this system would increase the interpretability of election results. In addition, the use of tie-breaking rules increases the richness of the vocabulary with which voters may express their votes.

In order to further reduce declared-strategy voting's susceptibility to manipulation, we must build into the system the ability to identify an optimal sophisticated strategy on a voter's behalf (should such a strategy exist). Whether this is possible for all sets of preference information and for all types of traditional aggregation procedures remains an open question. Farquharson [9] demonstrated an iterative elimination process for identifying optimal sophisticated strategies for binary voting procedures (procedures in which the alternatives are voted on two at a time, with winning alternatives advanced to the next round and losing alternatives eliminated until only one alternative remains). Felsenthal and Maoz [10] applied this procedure to plurality and approval voting. Further work is needed to determine whether this procedure is useful in the declared-strategy framework.

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