WASHINGTON UNIVERSITY

SEVER INSTITUTE OF TECHNOLOGY

DEPARTMENT OF ENGINEERING AND POLICY

DECLARED-STRATEGY VOTING: AN INSTRUMENT FOR GROUP DECISION-MAKING

by

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Prepared under the direction of Ron K. Cytron

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ABSTRACT

DECLARED-STRATEGY VOTING: AN INSTRUMENT FOR GROUP DECISION-MAKING

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The goal of this research is to determine whether *declared-strategy voting* (DSV) can be an effective tool for group decision-making. DSV is a novel group decision-making procedure in which preference is specified using voting strategies — first-order mathematical functions that specify a choice in terms of zero or more parameters. Three ideas motivate DSV: the possibility of taking advantage of electronic voting systems to achieve more beneficial methods of vote aggregation than are feasible with traditional voting systems, the desire to find an information-neutral voting system, and the desire to maximize a voting system's expressiveness. We present designs for two types of DSV systems: batch DSV and ballot-by-ballot DSV. We also develop a rational-strategy formulator that can determine optimal strategies for each voter given only the voter's cardinal utilities for each alternative. Our strategy formulator is based on the expected-utility model of voting and employs a novel technique for calculating pivot probabilities. We present the results of a DSV acceptability study as well as DSV simulations of several types of voting situations.

DSV may be applied to group decision-making situations in which a policy-maker must make a decision using a non-binding survey as a guide, as well as to situations in

which a large or small group must make a decision through a binding vote. We demonstrate that DSV can help a decision-maker analyze group preferences. DSV is useful for gauging strength of preference and level indifference, identifying sets of alternatives that act as mutual substitutes, and determining which alternatives voters would support if they knew their favorite was not a serious contender. We also show that DSV can be effective as a voting system. DSV allows voters to vote strategically, maximizing the effectiveness of their votes regardless of whether they have obtained information about the preferences of the rest of the voters or whether they know how to formulate optimal strategies. DSV also allows voters to express their sincere preferences, including preference intensities, without sacrificing the effectiveness of their votes.

to my parents, who taught me how to play the game

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Lorrie Faith Cranor

Washington University in Saint Louis
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Glossary

amendment procedure A voting procedure that pairs a proposal under consideration with the *status quo*. If a variation on the proposal is introduced it is paired with the proposal and voted on as an amendment prior to the final vote. If the amendment succeeds, then the amended proposal is eventually paired with the *status quo* in the final vote.

approval voting A voting procedure in which voters may vote for (approve of) as many of the alternatives as they wish.

Borda count A voting procedure in which each voter forms a preference ranking for all alternatives. Given n alternatives, each voter casts n votes for his or her most preferred alternative, $n - 1$ votes for his or her second most preferred alternative, etc. The alternative that receives the most votes wins.

cardinal utility A preference expressed in terms of intensity on a finite scale.

Condorcet criterion Holds for a voting system if it selects the Condorcet winner whenever one exists.

Condorcet efficiency Percentage of elections in which the Condorcet winner is selected (assuming there is a Condorcet winner).

Condorcet winner An alternative that beats or ties all others in a series of pairwise contests.

consistency Holds for a voting system if when the electorate is divided arbitrarily into two parts and separate elections in each part result in the same alternative being selected, an election of the entire electorate also selects that alternative.

coordinated macromanipulation Manipulation by coalitions.

cumulative vote A semiproportional voting system in which each voter is alloted a number of votes equal to the number of candidates to be elected. The voters may give one vote each to that number of candidates or accumulate their votes, allocating two or more to one candidate.

declared-strategy voting (DSV) A novel group decision-making procedure in which preference is specified using voting strategies.

deterministic Holds for a voting system if it always produces an election winner.

effectiveness The power of a ballot to influence the election outcome.

expressiveness The extent to which a voting system allows voters to voice their honest opinions without sacrificing the power of their votes.

independence from irrelevant alternatives Holds for a voting system if it always produces the same results given the same profile of ordinal preferences.

insincere vote A vote that is not sincere.

limited vote A semiproportional representation system similar to plurality voting, except voters are limited to casting a number of votes that is less than the number of candidates to be elected.

list proportional representation (list PR) A proportional representation system in which political parties prepare lists that include as many candidates as there are seats to be filled. The voters select the list for the party they support and deposit it in their ballot envelopes. After the votes are counted and the seats are allocated, each party selects from the top of its list the number of candidates corresponding to the number of seats won. Some variations on this procedure allow voters the option of crossing out candidates or selecting particular candidates from a party list.

majoritarian voting method A voting method that selects a winner based on pairwise comparisons between alternatives.

majority rule A voting system in which the alternative receiving more than half the votes wins.

manipulability Holds for a voting system in which voters may sometimes obtain a more preferred outcome by casting ballots not reflective of their true preferences.

micromanipulation Manipulation by individuals.

monotonicity Holds for a voting system if when a voter raises the valuation for a winning alternative it remains a winning alternative, and when a voter lowers the valuation for a losing alternative it remains a losing alternative.

multi-stage voting method A voting procedure that uses either different choice functions at different stages of the procedure, or one choice function iteratively on diminishing sets of alternatives.

neutrality Holds for a voting system if the system does not favor any alternative.

non-dictatorial Holds for a voting system if the election outcome is not determined entirely by a single individual.

NP-hard Cannot be solved in polynomial time.

ordinal utility A preference expressed in terms of relative position in a list of alternatives ordered according to preference.

Pareto optimality Holds for a voting system if when every voter prefers alternative x to alternative y, alternative y is not selected.

pivot probability The probability that a voter will be decisive in creating or breaking a first-place tie between two candidates (or for elections that select M alternatives, the probability that the voter will be decisive in creating or breaking an M^{th}–place tie).

plurality voting A voting procedure in which each voter votes for his or her single most preferred alternative (or when n alternatives are to be selected, voters vote for their n most preferred alternatives). The alternative that receives the most votes wins.

positional voting method A voting method that selects a winner based on information about individuals' preference orderings.

proportional representation system (PR) A voting system that promotes close correspondence between the proportion of supporters of each party (or other interest group) in the electorate and the proportion of seats held by each party in a governing body.

rational Holds for voters that take actions they believe are most likely to lead their preferred outcome — regardless of whether it is reasonable for them to prefer that outcome.

runoff election procedure A two-stage voting procedure in which the alternatives are first considered together, and then if no alternative receives the majority of votes, a majority rule election is held between the two alternatives receiving the largest numbers of votes.

semiproportional voting system A voting system with similar properties as the proportional voting systems, but resulting in somewhat less proportional representation.

sincere vote A vote such that for any two alternatives under consideration, the voter has given greater or equal weight to the more preferred of those alternatives.

single-stage voting method A voting method that selects a winner in a single round of vote tabulation.

single non-transferable vote (SNTV) A special case of the limited vote in which voters may specify only a single candidate on their ballots, regardless of how many seats are to be filled.

single transferable vote (STV) A voting procedure in which voters submit a preference raking over all the candidates. Any candidate that receives more than a certain number of first-place votes is elected. If the elected candidates receive more votes than necessary for election, their excess votes are distributed to the other candidates in accordance with the second-choice preferences of the voters. Once again, any candidate that receives more than a certain number of votes is elected and the redistribution procedure continues. If the redistribution procedures reaches a point where there are no more votes to be redistributed and there are still more elected positions to be filled, the candidate with the least votes is eliminated and the votes for that candidate are redistributed.

sophisticated vote A vote determined to be optimal through game theoretic analysis, which takes into account the likely strategies of the other voters; a type of strategic vote.

stability Producing the same outcome from a given set of ballots each time it is counted.

strategic vote A vote such that for any two alternatives under consideration, the voter has given greater or equal weight to the more preferred of those alternatives.

utilitarian voting method A voting method that requires voters to assign *utility* values to each alternative. These utilities are aggregated to determine the election outcome.

utility A preference; a measure of personal value or satisfaction.

voting strategy A plan made prior to an election that prescribes the course of action a voter should take given any contingency that can arise.

Chapter 1

Introduction and Overview

Before the 1950 [UK] election the following question was asked in a Gallup poll: Would you vote Liberal if you thought the Liberals could win? To this question, no less than 38 per cent replied 'yes' — 28 per cent more than then intended to vote Liberal and 29 per cent more than actually did so. That is to say, the votes of more than one-quarter of the entire electorate were determined less by what they themselves wanted than by what they guessed most of the other electors to want. Knowing that this was so, the larger parties ... devoted much of their efforts to the capture of potential Liberal voters, not by extolling their own virtues or by attacking any feature of the Liberal programme, but by persuading the electors that the Liberal candidates had no chance — 'a vote for the Liberal is a vote wasted'.

Enid Lakeman, 1974 [63]

Suppose that there were a voting procedure that offered a straightforward strategy to every voter, whatever his preference scale .... A straightforward procedure would have immense advantages. No voter would ever need to hesitate how to cast his vote: the outcome of the voting would never depend upon the voters' erratic predictions how others are going to vote. We might well be willing to tolerate considerable deviation from the fairest outcome for the sake of such advantages.

Michael Dummett, 1984 [40]

Situations often arise in which groups must make a decision between three or more alternatives. However, there is no uniquely optimal way to make such a decision. A wide variety of voting methods have been proposed: some take into account only each voter's first choice, some take into account complete preference orderings, and some take into account the intensity of preferences. There are many techniques for aggregating preference

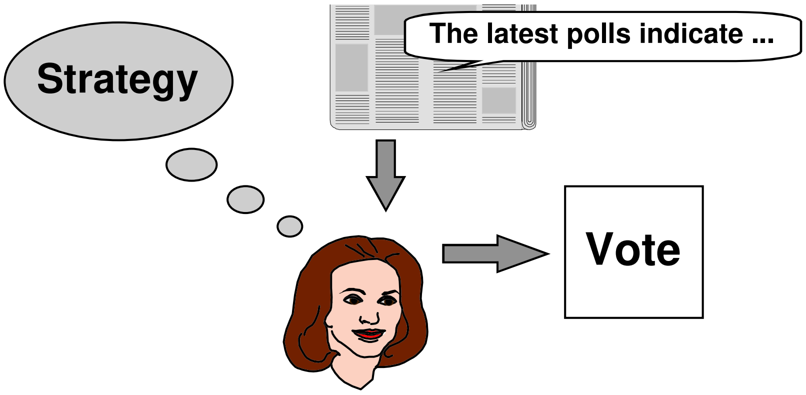

Figure 1.1: Voting strategies in traditional elections

information; often the election outcome is highly dependent on the voting method used. Advances in computer and telecommunications technology make feasible new techniques for collecting and aggregating ballots. In this dissertation we explore a novel group decision-making procedure made practical through such advances, and examine its ability to improve the decision-making process.

In a traditional election with three or more alternatives, some voters develop personal strategies to determine which alternative to select [24]. These strategies may take into account the voter's preferences for each candidate and the voter's estimate of the likelihood that each candidate will win the election. For example, voters may develop a strategy in which they vote for their most preferred alternative only if they believe that candidate has a "good chance" of winning. As illustrated in Figure 1.1, these voters may consider the results of pre-election polls to determine which alternatives meet their good-chance criteria before mentally evaluating their strategies. They express the result of this personal decision-making process in the form of a vote. However, traditional election results cannot distinguish between voters who vote for their sincerely preferred alternative regardless of other factors and those who base their votes on a strategy such as the one described above; this dilemma frustrates attempts at analyzing election results. In addition, inequalities in voters' access to information about the preferences of other voters and in knowledge about how to formulate optimal strategies may result in some voters having more power to influence the outcome of the election than their less informed peers.

We present *declared-strategy voting* (DSV), a novel group decision-making procedure in which preference is specified using voting strategies — first-order mathematical

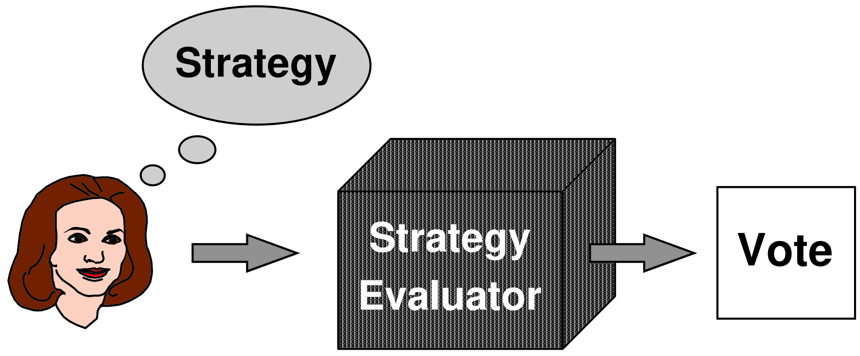

Figure 1.2: Voting strategies in declared-strategy elections

functions that specify a choice in terms of zero or more parameters. In a DSV system, illustrated in Figure 1.2, each voter submits a strategy that a computer resolves into a simple choice by substituting actual values for parameters. Parameters may include the percentage of counted ballots that select a particular candidate and the percentage of ballots already counted (or other measures of the "maturity of the election"). In Chapter 4, we present the design of an automated strategy formulator that will produce an optimal strategy for every voter from straightforward inputs.

When a decision must be made between three or more alternatives, the outcome can depend on the voting method used. For example, plurality voting selects the alternative most preferred by the most people, while Borda count voting tends to produce more of a compromise alternative (both of these procedures will be described in Chapter 2). But every non-dictatorial and deterministic voting method is subject to manipulation by voters who develop mental voting strategies and cast votes that misrepresent their *sincere* preferences. Declared-strategy voting dramatically reduces the opportunities for voters to benefit by misrepresenting their preferences, thus providing analysts with more accurate information about the true preferences of the electorate than can be obtained with traditional voting systems. This, in turn, may provide policy-makers with more accurate information on which to base their decisions. In addition, by minimizing the advantage gained by voters with access to the information needed to manipulate, declared-strategy voting provides voters with more equal opportunities to influence the election outcome.

Declared-strategy voting may be used in situations in which a single policy-maker bases a decision on the results of a non-binding survey, as well as in situations where a group makes a decision through a binding vote. Thus, the use of DSV may affect the ability of policy-makers to make the best decision, the ability of voters and survey respondents to express their preferences effectively, and the ability of analysts to interpret election results. DSV also impacts the result of the decision-making process itself, and thus affects all those for whom the decision is relevant.

The goal of this research is to determine whether declared-strategy voting can be an effective tool for group decision-making. This research focuses on exploring and refining the declared-strategy voting concept, assessing the potential impacts of declared strategy voting, and evaluating the effectiveness of declared strategy voting for group decision-making. This research makes several contributions:

- Throughout this dissertation we describe a new voting paradigm, *declared-strategy voting*. This paradigm enables voters to cast votes that are simultaneously effective and expressive. It also allows preference intensities to be considered in the vote aggregation process without relying on interpersonal comparisons of cardinal utilities.
- In Chapter 4, we introduce a new method for calculating pivot probabilities, the *2-D Gaussian* method. This method easily scales for use in elections with a large number of alternatives. It has applications not only for DSV, but also for other social science work involving pivot probability calculations.
- In Chapter 4, we also present an automated strategy formulator. This formulator is important for making DSV accessible to the general public. It may also be useful for voters who wish to vote strategically in traditional elections.

This dissertation is organized as follows. Chapter 2 provides background information about many types of vote aggregation and ballot collection systems and introduces terminology that will be used throughout this text. Chapter 3 explains the motivations for declared-strategy voting and the overall design of a DSV system. Chapter 4 presents the details of rational-strategy formulation. Chapter 5 evaluates DSV and presents some examples of its use. Chapter 6 presents our conclusions and proposes areas for future work.

Chapter 2

Voting Systems Overview

> *Voting is ... an indispensable feature of democracy because, however the goals of democracy are defined, its method involves some kind of popular participation in government. Although participation can take many forms, historically — and probably logically — it invariably includes voting.*
>
> William H. Riker, 1982 [86]

> *I believe that we cannot separate elections from governance. That is to say, innovative and inclusive governance must go hand-in-hand with more accountable, participatory electoral processes .... If campaigns are too negative, special interests too dominant, governance too partisan, decision-making too exclusive, we must ask whether these results are the logical outcome of the rules of our electoral game. And no rule is more fundamental than the voting system that translates votes into representation.*
>
> Cynthia Terrell, 1995 [98]

The choice of voting system can have a major impact on an organization or government. However, choosing a voting system is not a trivial task. Although many social-choice aggregation procedures have been proposed, none have been theoretically or empirically demonstrated to result in the "best" election outcome. Indeed, determining the best outcome has been shown to be impossible in many cases involving choices among three or more alternatives. In addition to choosing methods for aggregating voter preferences, organizations must also select methods for registering and authenticating voters, collecting ballots, and physically tallying the votes. In this chapter a wide variety of vote aggregation and ballot collection procedures are presented and discussed, and methods for evaluating these procedures are explored.

This chapter serves as an introduction to the vast array of voting systems that have been proposed. It is not intended to be a comprehensive catalog nor an indepth analysis, but rather it is intended to acquaint the reader with representative systems that highlight the approaches taken to preference aggregation and ballot collection. This chapter also serves to introduce terminology that will be used throughout this dissertation. These terms are also defined in the Glossary, which appears before Chapter 1.

2.1 Vote Aggregation Methods

> *The paradox of voting is the coexistence of coherent individual valuations and a collectively incoherent choice by majority rule. In an election with three or more alternatives (candidates, motions, etc.) and three or more voters, it may happen that when the alternatives are placed against each other in a series of paired comparisons, no alternative emerges victorious over each of the others: Voting fails to produce a clear-cut winner.*
>
> William H. Riker, 1982 [86]

The paradox of voting was discovered over 200 years ago by M. Condorcet, a French mathematician, philosopher, economist, and social scientist. However, it received little attention until Duncan Black [13] explained its significance in a series of essays he began in the 1940s. The importance of the voting paradox was not fully realized until several years after Kenneth Arrow published *Social Choice and Individual Values* [3] in 1951, which contained his General Possibility Theorem. The essence of this theorem is that there is no method of aggregating individual preferences over three or more alternatives that satisfies several conditions of fairness and always produces a logical result. Arrow's precisely defined conditions of fairness and logicality have been the subject of scrutiny by other scholars. However, none have found a way of relaxing one or more of these conditions that results in a generally satisfactory voting system immune from the voting paradox. Thus Arrow's theorem has the profound implication that in many situations there is no fair and logical way of aggregating individual preferences — there is no way to determine accurately the collective will of the people.

Social choice theorists have invented many vote aggregation systems and have attempted to determine the most appropriate systems for a variety of voting situations. Although there is some agreement about which characteristics are desirable in a vote aggregation system, there is much disagreement as to which characteristics are most important. In addition, the selection is often influenced more by political circumstances than by the advice of theorists. Thus the popularity of a voting system is not necessarily an indication of its fairness [66].

The choice of a vote aggregation system can influence much more than the results of an election. It can also influence the ability of analysts to interpret election results, and in turn the ability of representatives to understand the wishes of the people they represent and the satisfaction of the electorate that they have had the opportunity to express themselves. This is due to the fact that the various vote aggregation systems require voters to supply varying amounts of information about their preferences and that some systems tend to encourage voters to report their preferences insincerely. In addition, the choice of vote aggregation system could affect the stability of a government, the degree to which an organization embraces or resists change, and the extent to which minorities are represented. It could also affect the ability of the members of an organization to achieve compromise.

This section explores the many types of vote aggregation systems that have been proposed. Throughout this section the term *voting system* will refer specifically to the vote aggregation component of a voting system. Section 2.1.1 introduces criteria with which voting systems may be evaluated. Section 2.1.2 discusses *majority rule* voting and several *majoritarian* voting methods. Section 2.1.3 describes three *positional* voting methods. Section 2.1.4 introduces several *multi-stage* voting methods which are based on majoritarian and positional methods. Section 2.1.5 discusses several types of *utilitarian* voting methods. Section 2.1.6 introduces *proportional* and *semiproportional* voting systems, which are positional systems for electing multiple representatives in proportion to their support in the electorate. The vote aggregation systems described in this chapter are summarized in Figure 2.1, at the end of Section 2.1.1.

2.1.1 Evaluation Criteria

Many criteria have been proposed for evaluating voting systems. Because it is not possible to satisfy all criteria simultaneously, there has been much debate among theorists and political reformists as to which criteria are most important. Some of the most commonly cited criteria are introduced here.

The Condorcet Criterion

The *Condorcet winner* is an alternative that beats or ties all others in a series of pairwise contests. The Condorcet criterion holds for a voting system if it selects the Condorcet winner whenever one exists.

Voting systems that do not always select the Condorcet winner can be evaluated in terms of their *Condorcet efficiency* — the percentage of elections in which the Condorcet winner is selected (assuming there is a Condorcet winner).

Although the Condorcet criterion is a popular means for evaluating voting systems, there are some situations in which it is not clear that the Condorcet winner represents the collective choice. For example, Fishburn [46] presents a scenario in which five voters must choose between five alternatives (a, b, c, d, e). The voters hold the following preference orderings (listed from most preferred to least preferred):

voter 1:	$a\ b\ c\ d\ e$
voter 2:	$b\ c\ e\ d\ a$
voter 3:	$e\ a\ b\ c\ d$
voter 4:	$a\ b\ d\ e\ c$
voter 5:	$b\ d\ c\ a\ e$

Given this preference profile, a is the Condorcet winner. However, an examination of the number of times each alternative is ranked first, second, etc., bears the following results:

	a	b	c	d	e
Voters ranking alternative first	2	2	0	0	1
Voters ranking alternative second	1	2	1	1	0
Voters ranking alternative third	0	1	2	1	1
Voters ranking alternative fourth	1	0	1	2	1
Voters ranking alternative fifth	1	0	1	1	2

In an examination of rankings, b appears to be the best choice — with equal first ranks to a and more second and third ranks than a — yet the Condorcet winner is a.

Consistency

A voting system is *consistent* if when the electorate is divided arbitrarily into two parts and separate elections in each part result in the same alternative being selected, an election of the entire electorate also selects that alternative. If a voting system is not consistent than it may be manipulated through the establishment of strategically configured election districts.

The Implementation Criteria

The implementation criteria evaluate the ease with which voters may specify their preferences and election administrators may aggregate these preferences. The more information required of the voter, the more difficult the voting procedure from the voter's perspective. The more comparisons that must be done in the aggregation process, the more difficult the voting procedure from the administrator's perspective [77].

In cases where the electorate is educated, even the most complicated voting systems are unlikely to pose difficulties for most voters. However, a simple voting procedure may be very important when parts of the electorate are illiterate. In addition, regardless of the literacy of the electorate, any voting system that requires multiple rounds of balloting is likely to be problematic unless the electors are assembled for a meeting.

The advent of high-speed computers diminishes the significance of the number of comparisons necessary to count the votes. Once an important factor for determining the practicality of a voting system, this implementation criterion is now largely irrelevant in most situations where votes are tallied by computer. However, this criterion remains relevant for voting systems in which determining the winner is *NP*-hard[1] [9]. Another criterion related to system implementation is anonymity. Although most systems can be implemented with secret ballots just as easily as they can be implemented with non-secret ballots, some procedures require action on the part of the voter after initial balloting and thus may be difficult to implement with secret ballots. Cryptographic electronic voting (to be discussed in Section 2.2.4) and commerce protocols may provide solutions to this problem.

Independence from Irrelevant Alternatives

A voting system is *independent from irrelevant alternatives* if it always produces the same results given the same profile of ordinal preferences. This criterion has consequences in two areas: it prohibits voting systems that take into account cardinal preferences or ordinal preferences beyond each voter's first choice, and it prohibits voting systems that include elements of randomness [86].

[1] A problem is said to be *NP*-hard if it cannot be solved in polynomial time. If n is a variable describing the complexity of the problem (for example, number of voters or number of alternatives), we say that the problem can be solved in polynomial time if we can solve it in n, n^2, n^3, or another power of n steps. On the other hand, if the problem requires 2^n, 3^n, or another number raised to the n^{th} power steps to solve, it becomes quite intractable as n increases and is thus *NP*-hard [53]

To understand the first consequence, it is important to realize that it applies to subsets of alternatives. Thus if a group of voters reverse their preference ordering for two alternatives, no change should occur in the outcome with respect to another set of alternatives for which the profiles remain fixed. The same applies to changes in intensity of preferences. Thus voting systems that use cardinal preferences are not independent because a change in intensity of preference for a loosing alternative may render it a winning alternative, even if the voters who changed their cardinal preferences do not change their ordinal preferences.

The first consequence of the independence criterion can also be understood by considering Vickrey's definition: "The social choice between any two alternatives shall not be affected by the removal or addition of other alternatives to the field of feasible alternatives under consideration" [103]. Arrow gave as an example a situation in which a losing candidate dies after the ballots have been cast. If the voting system is independent, removing that candidate's name (and votes) from the ballots should not change the election winner [3]. If each ballot contains only the name of the voter's most preferred candidate (or an unordered set of preferred candidates), votes for the deceased would be eliminated and the vote totals for the surviving candidates would remain unchanged. However, if the ballots contain an ordinal preference ranking over two or more candidates, those ballots containing votes for the deceased would be recounted with the surviving candidates who are ranked below the deceased bumped up.

The fact that the independence criterion allows voting systems that use minimal information about voter preferences while prohibiting voting systems that take into account more information about voter preferences has made it controversial. Use of this criterion is generally motivated by the fact that it prohibits voting systems that use information that may be considered arbitrary (cardinal preferences) or that tend to promote strategic behavior. However, some theorists argue that in the absence of a perfect voting system, systems that require voters to supply cardinal preferences or ordinal preferences over multiple alternatives should not be ruled out because of a lack of independence [72, 103]. One might also argue that the systems that satisfy the independence criterion do so because they ignore information and essentially waste the votes of all those who do not support one of the top contenders. Thus those who support the "irrelevant" alternatives (those alternatives with little chance of winning) might as well not vote at all. Indeed these systems also promote strategic behavior because they tend to encourage supporters of the irrelevant alternatives to vote for second-choice alternatives with better chances of winning.

The second consequence of the independence criterion is to prohibit voting systems that do not have *stable* outcomes: that is, voting systems in which a given set of ballots may produce different results each time it is counted. Instability is rarely encountered in the traditional voting systems discussed in the chapter and has not been the subject of much discussion in the literature. It is generally taken for granted that voting systems should not involve random processes, except possibly to to resolve ties. Slaton offers an interesting explanation of this assumption using Newtonian and quantum physics as metaphors. She argues that traditional voting systems stem from a Newtonian world view in which it is believed that the world can be understood through the study of discreet particles whose behavior adheres to strict laws. She contrasts the Newtonian philosophy with the quantum philosophy, which places less emphasis on absolutes and more emphasis on tendencies. Random sampling is thus a reasonable approach to selecting representatives under a quantum world view [96].

Monotonicity

A voting system is *monotonic* if when a voter raises the valuation for a winning alternative it remains a winning alternative, and when a voter lowers the valuation for a losing alternative it remains a losing alternative. All voting systems that eliminate alternatives prior to selecting a winner violate monotonicity [86].

If is a system is not monotonic, a situation may arise in which voters could prevent their favorite alternative from winning by voting for that alternative. Obviously this is contrary to the purposes of a voting system — voters should not be penalized for supporting their favorite alternative.

Neutrality

A voting system is *neutral* if the system does not favor any alternative. Although it may seem obvious that voting systems should be neutral, some are designed specifically not to be neutral in order to promote government stability or ensure that a decision is made. In fact, many systems that have tie-breaking rules that do not employ a random choice violate neutrality. For example, parliamentary voting systems in which "yea" and "nay" votes are taken generally favor the nay side in the event of a tie. Thus they are biased towards the nay alternative and are not neutral.

Pareto Optimality

A voting system is *Pareto-optimal* if when every voter prefers alternative x to alternative y, alternative y is not selected. This criterion is similar to the monotonicity criterion, but is more often satisfied by the voting systems that have been proposed. It is sometimes referred to as the *unanimity* criterion.

Both Pareto-optimality and monotonicity relate to the collective rationality of a group decision-making procedure. Clearly if one were to make a decision based entirely on one's own preferences, one would not select y if one preferred x. The rational individual would select an alternative other than y (x or an alternative preferred to x). Likewise if one had selected x and later received news that x was even better than originally thought or that y was even worse than originally thought, one would not switch the selection to y. The rational individual would stick with the selection of x. When many decision-makers must make a group decision, we can extend the rules we would follow for individual decision-making. Thus if every decision-maker prefers x to y then it would be irrational for the group as a whole to select y. Likewise, if the group selects x and one or more group members decide that x is even better than originally thought or y is even worse than originally thought, the group should not change its decision. Thus with Pareto-optimality and monotonicity, the notion of what is individually rational can be extended easily to what is collectively rational.

Proportionality

Voting systems that elect multiple representatives may be evaluated in terms of the correspondence between the number of representatives elected from each party and the support for each party in the electorate. Some voting systems tend to favor larger parties or smaller parties, while others result in highly proportional representation. In addition to the voting method used, the number of representatives to be elected in each district can have a major impact on the proportionality of a voting system.

Consider the following example. A city council is composed of five representatives, each elected by the voters in a single-member district. Sixty percent of the city's voters are Democrats and 40% are Republicans. The distribution of party membership is fairly even across the five districts. Under this non-proportional representation system, all five representatives elected are Democrats. However, if the five representatives were elected at-large by voters from throughout the city using a proportional representation system, three Democrats and two Republicans would be elected.

Related to proportionality is the average ballot effectiveness. The effectiveness of an individual ballot may be defined as the proportion of a ballot that is used to vote for a candidate that ends up winning. In elections where each voter votes for one candidate and the candidate with the most votes wins, all ballots voting for the winning candidate are 100%

effective while the remaining ballots are 0% effective. The more proportional a voting system, the greater its average effectiveness is likely to be.

Tendency to Encourage Manipulation

Situations sometimes arise in which it is not always in the best interest of voters to vote for their most sincerely preferred alternatives — for example, in US Presidential elections with three or more candidates, supporters of a third-party candidate often vote for a second choice in order to avoid wasting their votes on a candidate with little chance of getting elected. When voters cast *insincere* ballots they are said to be voting *strategically* or *manipulating* the election. Gibbard [54] and Satterthwaite [94] proved that all non-dictatorial voting systems with at least three possible outcomes are manipulable. However, some voting systems tend to encourage manipulation more than others and situations in which it is advantageous to vote strategically arise more frequently in some systems than in others. Furthermore, it is easier for voters to identify advantageous strategies in some systems than in others. Many social-choice theorists have suggested that the tendency to encourage manipulation is an important criteria for evaluating voting systems [17, 40, 78].

Nurmi evaluates voting systems' tendencies to encourage manipulation in terms of the information required for successful manipulation. He sketches a four-level hierarchy for preference misrepresentation. At the first level of the hierarchy are those systems that require knowledge of only the "distribution of first preferences over alternatives" in order to be manipulated. At the second level are those systems that require knowledge of "distribution of approval over alternatives." At the third level are those systems that require knowledge of the "binary comparison matrix." And at the fourth and most difficult to manipulate level are those systems that require knowledge of the "entire preference profile" [78].

While Nurmi's hierarchy is helpful in identifying the relative difficulties involved in obtaining the information necessary to manipulate an election, it does not distinguish voting systems based on the ease with which this information may be applied or the chances that this information will reveal preference profiles over which manipulation is possible. Indeed, it has been shown that manipulating some voting systems is an *NP*-hard problem, even with perfect information about voter preference profiles [8]. Nurmi's hierarchy also fails to distinguish between systems manipulated most successfully by individuals (micro-manipulation) and those manipulated most successfully by coalitions (coordinated macro-manipulation).

Voting Procedure	Type	Always Selects C-Winner	Never Selects C-Loser	Consistent	Independent	Monotonic	Neutral	Pareto-Optimal	Manipulability
Amendment	Majoritarian	✓	✓	✗	✓	✓	✗	✗	III
Approval	Positional	✗	✗	✓	✓	✓	✓	✓	II
Black	Multi-stage/ Majoritarian/ Positional	✓	✓	✗	✗	✓	✓	✓	III
Borda	Positional	✗	✓	✓	✗	✓	✓	✓	III
Coombs	Muli-stage/ Positional	✓	✓	✗	✗	✗	✓	✓	IV
Cumulative Vote	Positional/PR	✗	✗	✓	✓	✓	✓	✓	
Limited Vote	Positional/ Semi-PR	✗	✗	✓	✓	✓	✓	✓	
List PR	Positional/PR	properties depend on procedure used							
Nanson	Multi-stage/ Positional	✓	✓	✗	✗	✗	✓	✓	III
Plurality (first past the post)	Positional	✗	✗	✓	✓	✓	✓	✓	I
Runoff (majority runoff, plurality run-off, second ballot, two-ballot, double ballot, exhaustive ballot)	Multi-stage/ Positional	✗	✓	✗	✓	✗	✓	✓	IV
Simple Majority Rule * with 3 or more alternatives	Majoritarian	✓	✓	✗	✓	✗	✗	✓	III
Single Nontransferable Vote	Positional/ Semi-PR	✗	✗	✓	✓	✓	✓	✓	I
Single Transferable Vote (Hare, preference vote, alternate vote, majority preferential)	Positional/PR	✗	✓	✗	✗	✗	✓	✓	IV
Successive	Majoritarian	✗	✓	✓	✓	✓	✓	✓	
Demand Revealing, Aggregation of Cardinal Utilties	Utilitarian	✗	✗		✗		✓		

Figure 2.1: Properties of vote-aggregation procedures. Check marks indicate properties possessed by the voting system in that row, Xs indicate properties not possessed by the voting system in that row. Cells have been left blank where information is unavailable or varies depending on the specific details of the procedure used.

2.1.2 Majority Rule and Majoritarian Methods

Majority rule is the simplest of non-dictatorial voting systems. When two alternatives are under consideration, the one preferred by the most voters is selected. However, when three or more alternatives are under consideration, another procedure must be used to narrow down the field of alternatives or ensure that one of the alternatives is preferred by the majority. Majority rule with two alternatives possesses all of the most desirable properties of voting systems. In fact, it is often considered the best method of preference aggregation [86].

Unfortunately, decisions can rarely be considered as a choice between two alternatives unless another mechanism has been used to eliminate all but two alternatives or divide the alternatives into two groups. The two-party system in the United States serves to narrow down the fields of candidates so that only two candidates compete for each office in most state and national elections. Parliamentary procedures generally involve an amendment protocol that results in all votes being binary choices between a new proposal and the *status quo* or a proposal and an amendment. But when runoff elections or amendment procedures are used to reduce the field of alternatives to two, alternatives may be eliminated that could actually win the election if an alternate procedure were used. Thus when majority rule is used with three or more alternatives, one or more criteria may be violated as a result of the procedure used to narrow down the alternatives. For example, Riker presents a scenario in which monotonicity is violated when 13 voters choose between four alternatives (w, x, y, z). The voters hold the following preference orderings:

voters 1, 2:	$w\ x\ y\ z$
voters 3, 4:	$w\ y\ x\ z$
voters 5, 6, 7, 8:	$x\ w\ y\ z$
voters 9, 10, 11:	$y\ z\ x\ w$
voters 12, 13:	$z\ w\ y\ x$

In a one-person-one-vote election, w receives four votes, x receives four votes, y receives three votes, and z receives two votes. If all but the top two alternatives are eliminated, w and x will compete in a simple majority runoff election. In this election voters 9 through 13 will each vote for their second choice, as their first choices have been eliminated. Thus w will receive six votes and x will receive seven votes. Now, suppose voters 1 and 2 change their minds and decide to reverse their preferences for x and w — increasing the overall support for x. In this case the runoff election will be a contest between x and y. Given that the support for x has increased, we might expect x to remain victorious. However, in this case y beats x seven votes to six. Thus the runoff procedure has caused monotonicity to be violated.

Simple majority rule may be applied to three or more alternatives by comparing the alternatives two at a time, eliminating an alternative if it loses a pairwise comparison. However, the order in which alternatives are selected for comparison may determine the outcome of the election. For example, given the above preferences, either w, x, or y might win, depending on what order they are selected for comparison.

Majoritarian methods extend the principles of majority rule for more than two alternatives by selecting the alternative that can beat all other alternatives in pairwise comparisons (the Condorcet winner). However, such an alternative is often non-existent, and these methods must provide additional procedures for selecting a winner in the absence of a Condorcet winner. Because these methods rely on binary comparisons between all candidates, voters must supply their complete preference orderings or the election must be conducted with multiple rounds of balloting [86]. Thus, knowledge of the binary comparison matrix is required for them to be manipulated [78].

Many majoritarian methods have been proposed, most of which are monotonic and result in the selection of the Condorcet winner when one exists. All violate the consistency criterion, and they have differing outcomes in the absence of a Condorcet winner. Except for the amendment procedure, these methods are rarely used. The following examples of majoritarian methods are described by Riker [86] and Nurmi [77].

The Amendment Procedure

The amendment procedure is widely used for parliamentary and board room voting. This procedure pairs the proposal under consideration with the *status quo.* If a variation on the proposal is introduced it is paired with the proposal and voted on as an amendment prior to the final vote. If the amendment succeeds, then the amended proposal is eventually paired with the *status quo* in the final vote. Otherwise, the amendment is eliminated prior to the final vote. This procedure is not neutral because it favors the *status quo* in the absence of a Condorcet winner.

The Successive Procedure

Unlike the amendment procedure, the successive procedure is neutral. However it does not select any winner in the absence of a Condorcet winner, and may sometimes select no winner even when a Condorcet winner exists. In this procedure the candidates are considered one at a time until one has received the majority of votes or the list of candidates has been exhausted. After the list has been exhausted the procedure may be repeated (with the hope that some of the voters may change their votes) or another procedure may be used.

Other Majoritarian Procedures

The Copeland rule selects the alternative with the largest Copeland index, which is the number of times an alternative beats other alternatives minus the number of times that alternative loses to other alternatives when the alternatives are considered in pairwise comparisons. The Copeland rule is Pareto-optimal.

The Schwartz rule selects the set of alternatives over which the collective preferences are cyclic and the entire cycle is preferred over the other alternatives. When there is a Condorcet winner it is the single element in such a set. However, in the absence of a Condorcet winner most or all of the alternatives may be members of this set. The Schwartz rule is not Pareto-optimal.

Other majoritarian procedures include the maximin method, Dodgson's method, and the Kemeny rule. Determining the winner using Dodgson's and Kemeny's procedures is NP-hard [9]. See [77, 78, 86] for descriptions of these procedures.

2.1.3 Positional Methods

Unlike majoritarian methods, which use only information from binary comparisons between candidates, positional methods take into account information about individuals' preference orderings. However, these methods do not necessarily take into account a voter's entire preference ordering. In this section, three single-stage positional methods are described: plurality voting, which uses only information about each voter's most preferred alternative; approval voting, which uses information about a varying number of top alternatives for each voter; and the Borda count, which uses information about each voter's entire preference ordering. These methods do not necessarily select the Condorcet winner when one exists. In addition, for some sets of voter preference profiles, each method produces a different outcome. The three positional methods described here are all monotonic and Pareto-optimal.

Plurality Voting

Plurality voting, also referred to as *first past the post,* is a procedure in which each voter votes for his or her single most preferred alternative (or when n alternatives are to be selected, voters vote for their n most preferred alternatives). The alternative that receives the most votes wins. Because this procedure only takes into account the first choice preference of each voter, it often fails to select the Condorcet winner. As a result it sometimes produces election results which appear illogical when three or more alternatives compete. It sometimes even selects the Condorcet loser, the alternative that is defeated by all others in pairwise comparisons.

In plurality elections with three candidates, two candidates with somewhat similar ideological views will sometimes both lose to a candidate with opposite views. However, after the election the supporters of the two losing candidates may realize that had they worked together and supported a single candidate, that candidate could have won easily. In the next election, these supporters are more likely to compromise and back a single candidate. Thus, over time, plurality voting encourages parties to offer only two candidates — reducing plurality voting to simple majority rule. Plurality voting is widely used for governmental elections in the United States and has been credited with helping to maintain the two-party system [86].

Plurality voting is classified at the first level of Nurmi's preference misrepresentation hierarchy and is generally considered to be one of the easiest voting systems to manipulate [78].

Approval Voting

Approval voting, independently developed by at least five sets of people in the mid-1970s, allows voters to vote for (approve of) as many of the alternatives as they wish. Generally, the winner of an approval voting election is the alternative that receives a plurality of votes [17].

Approval voting is currently used by several professional societies, including the Institute of Electrical and Electronics Engineers, the Mathematical Association of America, the American Statistical Association, and The Institute of Management Sciences. It tends to promote the election of moderate candidates and prevent situations in which the candidate elected is ranked last by those who did not vote for him or her. It also tends to encourage three or more alternatives to be considered in an election. If all voters approve of only their first choice alternatives, approval voting results in the same outcome as plurality voting.

Nurmi classifies approval voting at the second level of his preference misrepresentation hierarchy because it requires knowledge of the distribution of approvals over alternatives in order to be manipulated [78]. In addition, Brams and Fishburn assert that approval voting is less likely to be manipulated than plurality voting because it presents voters with fewer situations in which it is advantageous to vote strategically. In many situations where plurality voters would fare better by voting for a second choice, approval voters can maximize their voting power by voting for both their first and second choices [17].

As with plurality voting, approval voting does not necessarily select the Condorcet winner when one exists. However, approval voting has a higher Condorcet efficiency than plurality voting, selecting the Condorcet winner in all cases where it is selected by plurality voting as well as in other cases where it it may not be selected by plurality voting [17].

The Borda Count

Borda count voting, proposed in 1770 by the French mathematician Jean-Charles de Borda, is a procedure in which each voter forms a preference ranking for all alternatives. Given n alternatives, each voter casts n votes for his or her most preferred alternative, $n-1$ votes for his or her second most preferred alternative, etc. The alternative that receives the most votes wins [89].

This system, as well as other similar "scoring" or "point" methods, incorporate all voter preference information into the vote aggregation. However, as with the other positional voting methods, this does not always produce a logical result. In fact, the result of a Borda count election is often a function of the number of alternatives considered [86]. However, the Borda count will never choose the Condorcet loser [77].

The Borda count is one of the few single-stage voting methods that fails to satisfy the independence of irrelevant alternatives criterion. This failure may be observed, for example, in a three-alternative election when two alternatives are tied for first place. If one voter switches his or her preference ordering for one of the tied alternatives and the third alternative without switching his or her preference ordering for the tied alternatives, the tie may be broken. For the irrelevance of independent alternatives criterion to hold, a switch in preference ordering not involving the tie should not affect the tie [86].

Although the Borda count was initially criticized for its susceptibility to manipulation (before it was known that all non-dictatorial voting systems with at least three possible outcomes are manipulable) later analysts have found it to be a mixed bag. Saari showed that the Borda count is less susceptible to micromanipulation than many other voting systems [88, 89] while Chamberlin demonstrated that the Borda count is more susceptible to coordinated macromanipulation than plurality rule and other positional voting systems [27]. Nurmi places the Borda count and its variations (including its multi-stage variations) at the third level of his hierarchy, noting that although voters submit their entire preference profiles, only binary comparisons are necessary to determine the election result and thus the binary comparison matrix is all the information needed to manipulate the election [78].

2.1.4 Multi-Stage Methods

Multi-stage procedures use either different choice functions at different stages of the procedure, or one choice function iteratively on diminishing sets of alternatives. The following methods are representative of the multi-stage procedures that have been proposed. All of these methods are Pareto-optimal. In addition, they have the advantage that voters can often vote for a preferred candidate who has little chance of winning without "wasting" an opportunity to vote for a more viable second-choice candidate (of course situations still arise where it is in a voter's best interest to vote insincerely). Thus voters are likely to have fewer incentives to vote insincerely with multi-stage procedures than with single-stage procedures.

Black's Method

Black's method simply selects the Condorcet winner when one exists and the Borda count winner in the absence of a Condorcet winner. This method is monotonic and never selects a Condorcet loser.

The Runoff Election Procedure

The runoff election procedure (sometimes referred to as *majority runoff, plurality runoff, second ballot, two-ballot,* or *double ballot*) is a two-stage procedure. It requires the alternatives to be considered together first. If an alternative receives the majority of votes, it is selected. Otherwise a runoff simple majority election is held between the two alternatives receiving the largest numbers of votes. This method is not monotonic and may select an alternative other than the Condorcet winner when a Condorcet winner exists.

Nurmi considers this procedure relatively difficult to manipulate and classifies it at the fourth level of his preference misrepresentation hierarchy [78]. However, the fact that it is not monotonic indicates that when voters have the necessary information, they will find opportunities for manipulation frequently.

A variation of this procedure used to elect the French National Assembly eliminates all candidates who do not receive a specified threshold of votes on the first ballot. As the second ballot may contain more than two candidates, the winner is the one who receives a plurality of votes [67].

Another variation on this procedure is the *exhaustive ballot* procedure in which the alternatives with the least votes are eliminated one at a time in a series of elections. This procedure can be time consuming and result in decreasing voter participation with each round of balloting [63].

Nanson's Borda-Elimination Procedure

Nanson's procedure applies the Borda count iteratively, eliminating the lowest scoring alternative after each round. Unlike the regular Borda count, this procedure will select the Condorcet winner when one exists. It will never select the Condorcet loser; however, it is not monotonic.

Single Transferable Vote

Single Transferable Vote (STV), also referred to as the *Hare* procedure, was developed by Thomas Hare in England and Carl George Andrae in Denmark in the 1850s. In this system, voters submit a preference ranking over all the candidates. Any candidate that receives more than a certain number of first-place votes is elected. If the elected candidates receive more votes than necessary for election, their excess votes are distributed to the other candidates in accordance with the second-choice preferences of the voters. Once again, any candidate that receives more than a certain number of votes is elected and the redistribution procedure continues. If the redistribution procedures reaches a point where there are no more votes to be redistributed and there are still more elected positions to be filled, the candidate with the least votes is eliminated and the votes for that candidate are redistributed. This procedure is illustrated in Figure 2.2. When STV is used to elect a single alternative (redistribution of excess votes is not performed in this case, only redistribution of votes from eliminated candidates is necessary) it is sometimes referred to as *alternate vote* or *majority preference* [63, 89, 106].

There are many variations on this procedure with different methods for calculating the quota needed for election, different redistribution procedures, and different methods for resolving ties. The procedure used to elect the Cambridge City Council and School Committee uses the Droop threshold to determine the quota needed for election. Any candidate

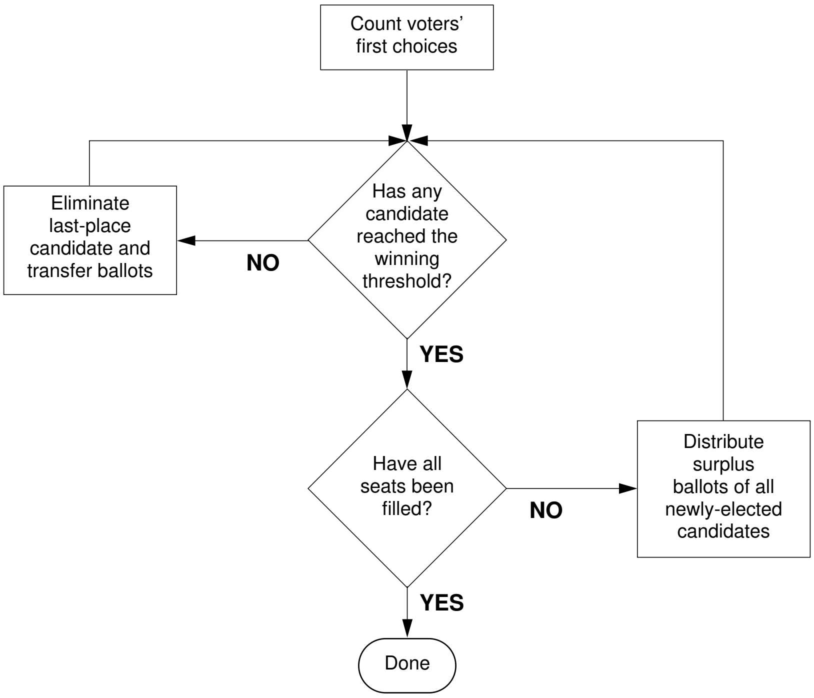

Figure 2.2: Procedure for counting votes in a single transferable vote election

receiving votes greater than the total number of votes divided by one more than the number of seats to be filled is elected. In Cambridge elections, votes are redistributed through a method of controlled randomness. For example if a candidate x receives 5000 votes and it is determined that only 4000 votes are needed to pass the Droop threshold, 1000 excess votes must be redistributed. The ballots for x are numbered and every fifth one is redistributed. (In Cambridge, ballots in each precinct are sorted according to the first choice listed. The piles of ballots for each precinct are merged according to a randomly determined precinct order.) While it is theoretically possible for this method to result in different election outcomes depending on the way the ballots happen to be ordered (due to second- or lower-choice alternatives being unevenly distributed among the ballots), a study by the Center for Voting and Democracy indicates that such outcomes are probably rare. The possibility of differing outcomes can be eliminated completely through the use of a mathematical redistribution system. In this system the excess ballots for candidate x in the above example would be distributed by redistributing one fifth of *every* ballot cast for x to the second choice indicated on that ballot. This method extends considerably the amount of time necessary for vote counting when ballots are tallied by hand. However, computerized vote-counting systems make mathematical redistribution practical [47].

The use of STV generally results in elections with high average effectiveness because all ballots that can be used to elect a candidate, either as a first-choice or on a transfer, have some degree of effectiveness. Because STV allows for redistribution of surplus votes when multiple candidates are to be elected, a single vote may be effective in helping elect more than one candidate.

Some theorists have criticized STV for its violation of several normative criteria. If the Condorcet winner is not eliminated in an STV election, it will be selected; however, it is possible for the Condorcet winner to be eliminated. Furthermore, this procedure is not monotonic and thus may result in perverse outcomes in which a candidate would be better off with fewer votes [18, 38]. In practice it is extremely difficult to detect violations of monotonicity in STV elections and thus such violations are rarely exploited or challenged [10].

STV requires knowledge of the entire preference profile to be manipulated and thus falls in the fourth level of Nurmi's hierarchy [78]. In addition, to the typical form of manipulation in which voters misrepresent their preference orderings, STV is also vulnerable to manipulation in which voters truncate their preference list and do not reveal their complete preference orderings. Despite the fact that examples of such vulnerability have been provided by Brams and Fishburn [18], literature explaining STV to voters continues to deny that truncation of preferences is ever advantageous [47]. This denial is probably of little consequence, however; as situations in which truncation or any other form of manipulation are advantageous in an STV election are almost impossible to detect [10]. But as Dummett explains, situations may arise in which parties may be able to coordinate successful macromanipulation. He proposes an example in which STV is used to select one of three candidates, a, b, or c:

> With detailed and reasonably accurate information about the intentions of the voters, such as can be obtained from well-conducted opinion polls, and with a thorough canvass to identify its own supporters, an organized group such as a political party can…instruct sufficiently many supporters to list a highest to ensure that a is not eliminated at stage 1, and instruct the rest to list b highest, in order to bring about the elimination of a's principal rival c [40].

STV is most vulnerable to such manipulation when it is used for elections in which one candidate is to be selected from a small pool of candidates. When multiple candidates are to be selected or when the pool of candidates is large, obtaining complete preference profiles for all voters and doing the computations necessary to find opportunities for manipulation is likely to be quite difficult.

Coomb's Procedure

Coomb's procedure is similar to STV; however, the alternatives with the most last place votes are eliminated (rather than the alternatives with the least first-place votes).

This procedure is not monotonic and does not always chose the Condorcet winner when one exists. However, it will not select the Condorcet loser. While STV tends to select the most intensely preferred alternative, Coomb's procedure tends to select the alternative viewed as least intolerable by the majority. It requires knowledge of the entire preference profile to be manipulated and thus falls in the fourth-level of Nurmi's hierarchy [77, 78].

2.1.5 Utilitarian Methods

Utilitarian voting methods require voters to assign *utility* values to each alternative. Utility values can be thought of as measures of expected happiness with each alternative on some finite scale.[2] These utilities are aggregated to determine the election outcome. Riker explains the rationale behind these methods:

> Majoritarian methods base decision on how many times x is ahead of one other alternative. Positional methods base decision on how many times x is ahead of all other alternatives. Neither bases decision on voters' direct valuation of alternatives, although positional methods are sometimes mistakenly so interpreted. Many writers, however, have argued that the intensity of judgment or direct valuation *ought* to be incorporated into the decision. The voting method should reveal not only that x is ahead of y, but by *how much* [86].

These methods do not necessarily select the Condorcet winner when one exists. They also violate the independence of irrelevant alternatives criterion. These methods do not fit cleanly into Nurmi's preference misrepresentation hierarchy because voters are likely to misrepresent the intensity of their preferences using these methods, even if they do not misrepresent their preference orderings. In order to manipulate the election successfully by misrepresenting their preference orderings, voters would need to know not only the entire *ordinal* preference profile required for Nurmi's fourth level systems, but also information from the *cardinal* preference profile. However, manipulation may be possible through misrepresentation of preference intensity without any additional information.

Several types of utilitarian methods described by Riker [86] and Mueller [72] are introduced below.

[2]The concept of utilities is theoretically grounded in the Von Neumann-Morgenstern method of determining cardinal measures of preference [104] However, in reality many people are unable to understand this method or use it to determine their utilities. Less precise estimates of utilities will often suffice.

Direct Aggregation of Cardinal Utility Methods

Several utilitarian methods involve simple mathematical aggregations of cardinal utilities. In the “summation of cardinal utility” method the utility values for each alternative are added and the alternative receiving the largest sum wins. In the “multiplication of utilities” method the utility values for each alternative are multiplied and the the alternative receiving the largest product wins. In some methods the utilities are fit to some normal scale before they are aggregated. These methods suffer from the fact that it is always in the voters’ best interests to inflate their preferences for their most preferred alternatives and deflate their preferences for their least preferred alternatives.

Demand-Revealing Methods

The demand-revealing method introduced in 1976 by Tideman and Tullock as “a new and superior process for making social choices” is one of the most unusual voting methods. The purpose of this system is to motivate people to state truthfully the values they place on public goods. The procedure operates as follows: Each voter offers to pay an amount of money m to obtain a preferred alternative. The amounts of money offered for each alternative are summed, and the alternative with the largest sum is the winner. In order to prevent people from offering more money than they really believe an alternative is worth, those whose offers for the winning alternative exceed the margin of victory must pay a tax equal to their contribution to the victory. This tax is wasted or given to people not involved with the vote so as to prevent its distribution from affecting the way people vote [99].

Riker criticizes this system for promoting inequality by providing an advantage to those whose budgets will be least impacted by paying a tax. He also criticizes it for encouraging coercion in an attempt to minimize the tax [86]. However, Tullock notes that the system need not use real money. Each citizen might be given a certain number of points which he or she may use for paying the voting taxes [101]. He does not comment on the possibility of markets developing for buying and selling these points. In addition, Riker notes that this system cannot be used in secret-ballot elections because it requires public revelation of individual offers in order to assess the taxes [86]. However, cryptographic electronic voting and payment systems might make such a system practical without public revelation of offers.

2.1.6 Proportional and Semiproportional Representation Systems

Proportional representation (PR) and semiproportional representation systems are designed to promote close correspondence between the proportion of supporters of each party (or other interest group) in the electorate and the proportion of seats held by each party in a governing body. To achieve this goal, these systems require the electorate to be divided into constituencies such that each elects more than one representative. There is a wide variety of methods used to achieve proportional representation, and each succeeds to varying degrees. Although on average PR systems produce results more proportional than produced by plurality voting systems, some PR systems are in reality less proportional than most plurality systems. Several countries have recently adopted mixed-member proportional representation systems (MMP) in which some representatives are elected from single-member districts and others are elected at-large using a PR system [33, 67].

Proportional representation was introduced in the nineteenth century as a way to provide representation for interests that could not be represented when a single representative was elected from each geographic community. After the breakup of the European empires at the beginning of the twentieth century, PR was selected by many countries in order to ensure that the educated minority would be represented in the new governments [6]

The many varieties of proportional representation systems that have been used to elect legislatures around the world can be divided into two categories: list PR (which employs a variation on plurality rule) and single transferable vote. The semiproportional systems all employ a variation on plurality rule. Although the goal of proportional representation might be pursued through adaptations of any of the positional or majoritarian voting methods, only plurality rule and STV have been used in practice.

List Proportional Representation

List PR is one of the most commonly used voting systems for governmental elections; however, it has never been used in the United States [6]. Although list PR is based on a simple idea, the implementation details of list PR systems are often complicated. These details involve the method used to determine how many seats should be allocated to each party and the method used to determine which individual will fill each seat. In all cases, the underlying decision rule is a variation on plurality rule.

In the simplest case, the entire country is treated as a single constituency, and the legislative seats are allocated in exact proportion to the support for each party among the electorate. However, even in this simple case, the mathematics of the situation may preclude the achievement of perfect proportionality. A country of population P represented by L legislators would ideally select one representative for each P/L voters. But it is unlikely that the vote totals for each party will be exact multiples of P/L, and fractional representatives are not an option. Several methods have been developed to solve this problem both in the case of a single constituency country and in the more usual case where a country is divided into many constituencies.

One of the easiest ways to allocate seats is to establish a quota necessary to win each seat. In this *largest remainder* method the parties are first allocated one seat for each multiple of the quota they have received. All votes received beyond an even multiple of the quota are a *remainder.* The remaining seats are allocated to the parties with the largest remainders. There are also more complicated methods, such as the *largest average* rule, that tend to have more proportional results [63].

The methods used to determine which individual will fill each seat vary in the extent to which they emphasize the influence of parties versus the opinions of individuals. The simplest forms leave the decision entirely up to the party while more complicated systems respect voters' opinions to varying degrees.

In the basic list systems, the parties prepare lists that include as many candidates as there are seats to be filled. The voters select the list for the party they support and deposit it in their ballot envelopes. After the votes are counted and the seats are allocated, each party selects from the top of its list the number of candidates corresponding to the number of seats won. A party winning one seat would select the candidate at the top of its list, a party winning two seats would select the top two candidates, and so on. The candidate at the bottom of the list has almost no chance of being elected [63].

Some countries allow voters to vote either for a party list or for a particular candidate on the list. In Belgium, for example, the vote tallies are added up for all of the candidates who received individual votes. All that receive enough votes to meet the quota are elected. The votes for the party list are then divided among the candidates on that list as follows: The first candidate on the list takes as many votes from the party list pool as are needed to meet the quota. If any votes remain in the pool the next candidate on the list takes as many as are needed to meet the quota, and so on until there are no more votes left in the pool. The more voters who choose to vote for the party list, the more influence the party has on the election result. In practice, the percentage of voters who do not vote for the party list (or take advantage of election laws in some countries that let them cross out candidates on their party list) is generally small [63].

Single Transferable Vote System of Proportional Representation

The single transferable vote system of proportional representation, also referred to as the *quota preferential* system and the *preference voting* system, uses the STV procedure (described in Section 2.1.4) or one of its variations to promote proportional representation in each constituency. STV has been used in Ireland for most of the twentieth century. It is also used in Malta, Australia, and Cambridge, Massachusetts. Unlike the list PR systems, which are focussed on supporting political parties, STV systems are designed to promote proportional representation by maximizing the effectiveness of every vote, regardless of whether the vote supports a particular party. In practice, this system often results in parties obtaining proportional representation, but this occurs only as a consequence of the voters choosing to vote along party lines [63, 67].

Limited Vote

The limited vote, a semiproportional representation system used in Spain and Portugal in the 19th and early 20th centuries and in parts of the United Kingdom in the 19th Century, is a plurality rule procedure that elects multiple candidates. However, unlike traditional extensions of the plurality rule for multiple candidates in which each voter may vote for as many candidates as are to be elected, this procedure limits voters to a number of votes less than the number of candidates to be elected. The number of votes is limited to encourage the representation of both majority and minority parties. Under traditional plurality systems, the majority party often supports as many candidates as are to be elected, and party members vote for all the party candidates. Thus it is difficult for minority candidates to be elected. However, in a limited vote system majority party members must either back fewer candidates than are to be elected or split their votes. This gives minority candidates a greater chance of being elected. It also tends to encourage parties to support fewer candidates than would be appropriate given the proportion of the electorate that supports that party. This is due to the fact that if parties over estimate their support, the vote for their candidates will be spread too thinly and most or all of their candidates will lose [63, 67].

Single Non-Transferable Vote

The single non-transferable vote (SNTV) is a special case of the limited vote in which voters may specify only a single candidate on their ballots, regardless of how many seats are to be filled. This semiproportional method has been used in Japan throughout most of the twentieth century [67]. It is credited with producing results more reflective of the wishes of the voters than are produced by other systems for electing multiple candidates. Because of its simplicity it is well suited for elections in which large parts of the electorate are illiterate. However, SNTV also tends to "cause stagnation, each party hesitating to run the risk of nominating more candidates than were successful in the previous election." It may also cause resentment among voters who do not feel that the system lets them express their preferences fully [63].

Cumulative Vote

The cumulative vote, a semiproportional system used to elect members of the Illinois State House of Representatives in the first half of the twentieth century, allots each voter a number of votes equal to the number of candidates to be elected. The voters may give one vote each to that number of candidates or accumulate their votes, allocating two or more to one candidate. In Illinois, when voters were to select three candidates they could give "one vote to each candidate, or one and a half votes to each of two candidates, or three votes to one candidate" [63]. This system has also been used for British school board elections and for electing corporate boards of directors. The cumulative vote system has many of the same advantages and disadvantages as the limited vote and SNTV systems. In addition, it allows more opportunities for voters to express themselves, but may be too complicated for some voters to understand [63].

2.2 Ballot Collection

The manner in which ballots are distributed, voted, collected, and counted is often as important as the system used for aggregating voter preferences. The system used can impact the overall voter turnout, the accessibility of the polls to all members of the voting population, the opportunity for fraud and vote tallying errors, the cost of conducting elections, the amount of thought voters put into their vote choices, and the electorate's confidence in the accuracy of the election results.

While a wide variety of voting systems and protocols exists, the basic procedure for conducting a democratic election is fairly standard. This procedure generally involves four tasks: registering voters, validating voter credentials, collecting voted ballots, and tallying the votes. This discussion will focus on the vote collection task, as it is most visible to the voters, and the protocols for performing the other tasks are often dependent on the way ballots are collected. Throughout this section, the term *voting system* will refer specifically to the ballot collection system.

Traditionally, governmental elections have been held at centralized precinct polling places, using hand- or computer-tallied paper ballots, mechanical voting machines, or direct-recording computer terminals. Large organization elections and recently some government elections are conducted via the postal mail. The possibility now exists for elections to be conducted via telephone, electronic mail, or the World Wide Web [35] as well. In this section some of the issues associated with each of these types of ballot collection systems are discussed.

2.2.1 Desirable Properties of Ballot Collection Systems

There are many properties that ballot collection system designers might try to incorporate into their systems. The following properties are generally desirable, but not necessarily obtainable in all types of systems. These properties were assembled following several similar property lists found in the literature [11, 48, 80, 92, 95].

Accuracy

A system is accurate if (1) it is not possible for a vote to be altered, (2) it is not possible for a validated vote to be eliminated from the final tally, and (3) it is not possible for an invalid vote to be counted in the final tally.

In the most accurate systems the final vote tally must be perfect, either because no inaccuracies can be introduced or because all inaccuracies introduced can be detected and corrected. Partially accurate systems can detect but not necessarily correct inaccuracies. Accuracy can be measured in terms of the margin of error, the probability of error, or the number of points at which error can be introduced.

Convenience

A system is convenient if it allows voters to cast their votes quickly, in one session, and with minimal equipment or special skills.

The convenience of a system is somewhat subjective, and depends not only on the system itself, but also on the voter population that will be using the system. For example, some populations might find it more convenient to vote at a centralized polling place using traditional voting methods while others might find it more convenient to vote from a personal computer in their homes or offices. For some populations it might be practical to locate centralized polling places at locations convenient to all voters; for other populations it will not be practical to do so.

Flexibility

A system is flexible if it allows a variety of ballot question formats including open-ended questions (this is important for write-in candidates and some survey questions).

Mobility

A system is mobile if there are no restrictions (other than logistical ones) on the location from which a voter can cast a vote.

Systems that require votes to be cast at designated polling places are not mobile unless they have provisions for absentee ballots.

Privacy

A system is private if (1) neither election authorities nor anyone else can link any ballot to the voter who cast it, and (2) no voter can prove that he or she voted in a particular way.

The second privacy factor is important for the prevention of vote buying and extortion. Voters can only sell their votes if they are able to prove to the buyer that they actually voted according to the buyer's wishes. Likewise, those who use extortion to force voters to vote in a particular way cannot succeed unless they can demand that voters prove that they voted as requested. Benaloh and Tuinstra explain:

> There are reports that in some small Italian villages, the voting system employed allows voters to list their votes in any order. Political bosses are said to assign different permutations of their preferred candidates to each voter. If a particular permutation fails to appear when the votes are counted, a boss can assume that the voter to which that permutation was assigned did not vote "properly", and reprisals can be taken [11].

Verifiability

A system is verifiable if anyone can independently verify that all votes have been counted correctly.

A weaker definition of verifiability used by some authors [48, 92] allows that a system is verifiable if it allows voters to verify their own votes and correct any mistakes they might find without sacrificing privacy. Less verifiable systems might allow mistakes to be pointed out — but not corrected — or might allow verification of the process by party representatives but not by individual voters. Traditional voting systems generally only allow for minimal verification by party representatives.

Vulnerability

A system is invulnerable, e.g. to ballot box stuffing, if (1) it permits only eligible voters to vote, and (2) it ensures that each eligible voter can vote only once.

2.2.2 Traditional Polling in the United States

In the United States, most national, state, and local voting takes place at centralized precinct voting places. The vote-collecting procedures vary greatly from precinct to precinct. Although some small precincts still use hand-tallied paper ballots, most precincts now use mechanical voting machines, direct-recording electronic voting machines, or machine-readable ballots.

Mechanical voting machines have a series of levers that voters pull to register their votes. Upon exit from the voting booth, the levers physically increment mechanical rotary counters. When the polls close, the backs of the voting machines are opened to reveal the vote tallies.

Machine-readable ballot systems, in use since 1964, require voters to mark their votes on a paper card with a pencil or marker, or remove divots from a perforated card with a stylus or mechanical hole puncher. The ballot cards are generally taken to a centralized computer center where they are scanned and tallied.

Direct-recording electronic voting machines require voters to use a keyboard, touch screen, or pointer to mark their ballot on a computer terminal. The votes are immediately added to a running tally.

Most traditional election systems are far from ideal. They tend to rely on a number of trusted parties who have the ability to conspire to change the outcome of the election or reveal the way particular voters voted. These systems generally work because most of the trusted parties are either trustworthy or have little trust in each other, and thus no conspiracy takes place. Although there has been much suspicion about security problems associated with computerized vote-tallying systems, there have been no proven cases of election fraud achieved through tampering with or manipulating such systems. All documented inaccuracies in computerized vote-tallying have been the result of problems with or misuse of the voting equipment or software. For example, a 1984 Carroll County, Maryland school board election was incorrectly tallied by a computerized tallying system after an election administrator accidentally installed the wrong utility program for reading ballot cards [39, 60, 93].

Experts disagree on whether it would be possible for a national election to be thrown by manipulating the computerized vote tallying system. Some see this as a real possibility, while others say it would be difficult for a national election to be thrown because of the large number of precincts (over 10,000) and the diversity of voting systems used. Most experts recommend that improvements be made to the vote tallying systems in widespread use and that selected precincts be tallied by hand to spot check the accuracy of these systems [39].

Regardless of the possibilities of tampering, electronic vote counting systems are attractive because they can count votes quickly and with minimal staff requirements, saving time and money. For example, *Computerworld* reported that the state of Oklahoma saved $1 million by introducing computerized vote counting in 1992 [56].

The punch-card system, the most widely used electronic vote tallying system, has been heavily criticized for its susceptibility to tampering and for its difficulty of use. Unlike direct-recording systems and mechanical voting machines, the punch-card system does not provide any automatic checks that the correct number of votes were cast for each office on an individual's ballot. In addition, if voters are not careful, paper bits called "chad" [3] can remain attached to the back of the punch card and work their way back into the holes that were punched. In 1987 it was found that ballots cast in predominantly black wards in the city of St. Louis were more than three times as likely to be improperly punched — and therefore not counted — as those cast in predominantly white wards. A federal judge subsequently ruled that the punch card system "denies blacks an equal opportunity with whites to participate in the political process." The judge ordered the city to increase voter education in black wards and count improperly marked ballots by hand [39].

Traditionally, the accuracy of vote tallies has been verified by party observers. However, it is not possible for voters to verify that their individual ballots were counted correctly. When votes are tallied via computer or mechanical voting machine it often becomes more

[3] *The New Hacker Hacker's Dictionary* explains a possible origin for the word chad:

> One correspondent believes *chad* ... derives from the Chadless keypunch (named for its inventor), which cut little u-shaped tabs in the card to make a hole when the tab folded back, rather than punching out a circle/rectangle; it was clear that if the Chadless keypunch didn't make them, then the stuff that other keypunches made had to be 'chad' [84].

difficult to verify that votes are counted properly and that votes are not being systematically changed, added, or eliminated.

The use of absentee ballots gives national elections the mobility property, allowing voters to cast their votes from almost anywhere they want. However, absentee ballot systems tend to reduce privacy and increase the opportunity for ballots to be changed or lost.

2.2.3 Vote-by-Mail

Vote-by-mail balloting has long been used by large geographically distributed organizations. It has also been used by states for absentee balloting and for special elections and elections involving small precincts. Recently there has been increased interest in using vote-by-mail more widely in state elections. Vote-by-mail supporters claim this type of balloting usually results in increased rates of voter participation, less expensive elections, higher levels of election integrity, and increased accessibility for the elderly and disabled [4, 45, 81]. Opponents are critical of the levels of security and privacy associated with vote-by-mail balloting and sometimes suggest that the ease with which people can cast a ballot by mail makes them take the electoral process less seriously [23, 61].

Large professional, social, and special-interest organizations often hold their elections through simple vote-by-mail balloting systems. These systems are convenient for voters and vote talliers; however, they often sacrifice privacy because voters must write their names or identification codes on their ballots. Although these systems generally incorporate minimal security measures there are usually few problems because organizations that use them tend not to hold highly controversial elections. Some organizations hire a disinterested party (generally at considerable expense) to run their elections so as to prevent election fraud and obtain the staff support needed to process the ballots quickly.

Organizations using vote-by-mail systems need not sacrifice voter privacy or security. Voter privacy can be protected through the use of double-ballot envelopes and fraud can be prevented through careful tracking of ballots and auditing of the vote-counting process. For example, in April 1995, Fort Collins Colorado first used a PC-based ballot verification program called Voyager to track ballots in a vote-by-mail election. The city staff scanned bar codes on the ballot envelopes as they were received, enabling the system to track the return of ballots and expose voters' attempts to vote multiple times. Fort Collins Mayor Ann Azari reported that the vote-by-mail election resulted in "increased voter turnout, compliance with ADA [Americans with Disabilities Act] requirements, elimination of the need to train and manage over 250 election judges and alternates — and concerns about insurance coverage for 83 voting locations went away" [4].

Indeed, with appropriate precautions, vote-by-mail can be suitable for even the most controversial elections. Probably the largest non-governmental organization to use vote-by-mail balloting to date is the Teamsters. In 1988 the Teamsters Union sent mail ballots to 1.5 million members. According to Teamsters election officers, the only problems encountered were a few attempts to vote multiple times or intimidate voters [73].

Many states use vote-by-mail balloting for some elections, especially in small precincts. For example, in the 1970s small precincts and water districts in California began using vote-by-mail balloting. Officials credited vote-by-mail with substantially increasing voter turnout while decreasing the costs of running these elections [73]. The state of Oregon began conducting some elections by mail in 1981. The first of these elections had an average voter participation rate of 78.1% and a cost of $.90 per vote cast, while the other elections held in the state that day had an average voter participation rate of 25.6% and cost $1.05 per vote cast. Subsequent vote-by-mail elections resulted in similar voter participation rates and cost savings [81]. Impressed by these results, law makers in other states began enacting laws permitting vote-by-mail balloting. By 1992 vote-by-mail elections were legal in 13 states [73].

In December 1995 Oregon held a vote-by-mail primary election to nominate candidates for a U.S. Senate vacancy, thus becoming the first state to conduct a mail-only primary election to nominate candidates for a federal office. Oregon held the general election for the vacant Senate position by mail in January 1996, and the state's Presidential preference primary by mail in March 1996. Citizen reaction to these vote-by-mail elections was generally favorable. A survey conducted by DRC Opinion Research in March 1995 found that 29.6% of the 604 respondents opposed using vote-by-mail balloting in the May primary and November general elections. Only 19% opposed using vote-by-mail balloting for these elections after they were informed that the state was likely to save $2 million for each election conducted by mail [85]. A telephone survey conducted by the Oregon Survey Research Laboratory following the January election found that 76.5% of the 1225 respondents "favored vote-by-mail elections over polling place elections" [97]. Another telephone survey of 1483 Oregon citizens found that respondents who indicated they had stamps in their home at the time they were interviewed were more likely to prefer vote-by-mail to polling place elections and more likely to describe vote-by-mail as convenient [100]. This suggests that some of the remaining opposition to vote-by-mail in Oregon is due to the fact that the state requires voters to pay their own return postage or hand deliver their ballots.

Besides the convenience and cost savings of vote-by-mail elections, some supporters also credit them with promoting "cleaner campaigns, less demagoguery and maybe more intelligent discussion of the candidates and the issues" [45] because voters have more time to examine their ballots and a longer time window (generally two to three weeks) in which to vote. Thus voters do not feel rushed to make their decisions and candidates gain little benefit from last minute attacks against their opponents.

Nonetheless, some people are skeptical still about the security of vote-by-mail balloting. The California and Kansas Supreme Courts have both ruled on cases involving vote-by-mail balloting. In both cases the courts refused to strike down laws allowing vote-by-mail balloting, despite the Kansas court acknowledging that "vote by mail increases the potential for compromise of secrecy and opportunity for fraud" [73]. Although media commentary on vote-by-mail has been mostly positive, some media organizations have expressed concerns. In December 1995, a Portland, Oregon radio station editorial criticized vote-by-mail for its lack of privacy and because it contributes to "the loss of the voting ritual." The station argued:

> If you're voting at home, a spouse may try to influence who you vote for and no one will report that. We've also been experiencing a wave of neglect and crime against the elderly by adult children. Influencing their vote would fit right in and there's nothing in the law that bans voting together with others at church or social clubs and there could be peer pressure to vote a certain way [61].

Bill Lunch, an Oregon State University political science professor agreed, "If there are problems out there, we are not likely to hear about them because they involve husbands or wives or other family members trying to exercise undue influence on each other" [23].

Oregon Secretary of State Phil Keisling dismisses concerns about vote-by-mail systems as pure speculation: "As of now, we haven't has a single allegation of fraud or undue influence. A lot of concerns that have been raised have turned out to be the figments of some people's imagination" [23]. Indeed, in a telephone survey conducted after the January 1996 vote-by-mail election, only four out of 1483 respondents indicated that they were not alone when they voted and that they would have voted differently if they had been alone [100].

Polls conducted after each Oregon vote-by-mail election held between 1981 and 1983 showed "no undue influence to pressure a person to vote a certain way" [81]. In fact, then Secretary of State Norma Paulus reported that vote-by-mail resulted in "a significant increase in the integrity of elections" due to the fact that voter signatures could be carefully verified, and ballots sent to voters who had moved without updating their registration were returned by the post office. In the first Oregon vote-by-mail election, 20.8% of the 25,913 ballots mailed were undeliverable and 127 ballots were rejected due to missing signatures or signature discrepancies. After investigation, six cases were referred to the attorney general for possible prosecution. In contrast, the challenge procedures used to detect vote fraud in traditional polling place elections are "virtually ineffective" [81].

2.2.4 Electronic Voting by Phone and Networked Computers

…Democracy must be structurally modernized
must be mechanically implemented
to give it a one-individual-to-another
speed and spontaneity of reaction
commensurate with the speed and scope
of broadcast news
now world-wide in seconds.

R. Buckminster Fuller, 1940 [49]

Futurist Buckminster Fuller was perhaps the first to suggest that voting be conducted electronically when he proposed "electrified voting" in his 1940 essay *No More Secondhand God* [49]. He wrote: "This is a simple mechanical problem involving but fractional effort of that involved in distributing the daily mails to the nation." Although he gave no details as to how this was to be accomplished, his reference to telephone calls in the next paragraph of his essay suggests that he envisioned a telephone voting system.

Despite Fuller's optimism about the simplicity of building such a system, the task remained impractical until the advent of computers. In 1973 Vincent Campbell and Janet Santos demonstrated a "televoting" system by conducting a series of surveys for the San Jose Unified School District. Over 5,500 of the 180,000 people living in the district registered to participate in this National Science Foundation funded project. Voters were mailed information regarding each televote question along with voting instructions and secret televoter numbers. They were asked to call the televote computer and enter their televoter number and a three-digit code corresponding to the response most reflective of their opinion on the question at hand. A "small eight-bit computer" transferred the televotes onto magnetic tape. At the end of the week the data on the magnetic tapes was transferred via telephone to a commercial data processing firm so that the results could be tallied. The investigators estimated that for cities with populations of 50,000 to 500,000 people, a televote system would cost \$25,000 to \$30,000 to install and \$.15 to \$.50 per capita yearly to operate [25]. Although the demonstration proved successful, there was little interest in telephone voting until the 1990s.

In the Fall of 1992, Sandia National Labs conducted experimental vote-by-phone elections at four New Mexico high schools. Over 1500 students submitted mock votes via telephone. Afterwards, 95% of the students who tried the system said they would use such a system to vote in real elections if it were available. Mary Green, security specialist for high-risk applications at Sandia National Labs, told a Congressional committee that researchers had learned three things from this experiment:

> (1) phone voting *can* be designed to be simple to use ...; (2) the volume of calls could overload your phone network so the period of phone voting would need to be extended; and (3) there are many ways to break into or sabotage such a system and only a few of these vulnerabilities have ready solutions [102].

Green outlined 10 objectives for secure phone voting and noted that the extent of the security risk varies with the type of election. She suggested that a secure phone voting system be "designed such that, in order to break into it, an adversary would be forced to spend so much money and manpower that he could have 'bought' all his voters or paid off elected officials for less money, less effort, and with less risk." She recommended that the U.S. pursue phone voting with extreme caution so as to prevent problems that might cause phone voting to "be discredited for the next 30 years" [102].

Other groups that tried phone voting in 1992 would have benefited from following Green's advice. A call-in poll conducted by CBS Television News and AT&T following George Bush's State of the Union Address on January 28, 1992 was the first test of large-scale telephone voting. This test proved problematic as only 314,786 of the 24.6 million calls to the vote-tallying computer got through. On June 6, 1992, the Nova Scotia Liberal Party held a province-wide election via telephone with the help of Maritime Telephone and Telegraph (MT&T). The election was held in conjunction with the party's conventions, with speeches televised for those who were not in attendance. The election was disrupted when MT&T's computer crashed and no backup systems were readily available. Adamson *et al.* suggest that the problems might have been avoided had MT&T or the Liberal Party researched the CBS experiment before proceeding [1].

Following the June 6 fiasco, MT&T compensated the Liberal Party financially for the failed election and the party agreed to a second attempt at vote-by-phone. For the second election, held on June 20, MT&T had a bank of operators standing by to record the vote manually should the computer system fail. They had also guaranteed their system by posting a $350,000 performance bond. This time everything went smoothly and by the end of the day almost 14,000 votes were collected and tallied in two rounds of balloting [1]. The voter participation rate was four times greater than it had been for the party's previous election, which had been conducted via traditional means at a party convention [58].

On September 11, 1993, the Liberal Party of British Columbia held a vote-by-phone election. This election also used MT&T's televoting system and proceeded without any technical difficulties. Surveys conducted after the Nova Scotia and British Columbia elections indicate that a large majority of voters prefer the televoting system to paper ballots, delegate conventions, and other voting methods. A majority of those surveyed believe that televoting is more convenient, cost effective, and easier to use than paper balloting. The majority of registered voters surveyed said televoting was more private and secure than paper balloting, while the majority of those surveyed who were not registered to vote felt that paper balloting was more secure and private. Critics expressed some concern that candidates might buy voters' secret access numbers so that they could cast large blocks of ballots. However, only 9% of the voters surveyed indicated that someone else had voted on their behalf, and (although it was not requested) some provided explanations that "anyone would agree were innocent" [15].

Following the successful Liberal Party televotes, MT&T created a wholly-owned subsidiary, MT&T Technologies, to develop and market telephone voting and surveying services. The company established a high-security "Teledemocracy Centre" in Halifax, Nova Scotia, which houses all of the computer equipment necessary to run a vote-by-phone election and provides accommodations for personnel who must be sequestered for the duration of an election. The Teledemocracy hardware currently includes a series of IBM RISC System/6000 processors and 480 telephone lines [71].

Despite the success of telephone voting in Canada, this technology has been relegated to media polls and pay-for-vote publicity stunts in the United States. William Kimberling, a Federal Elections Commission official, told the *Wall Street Journal* in 1992 that the public was not ready for telephone voting. "The futuristic stuff is good for people who watch Star Trek regularly, but not my mother," he stated [58]. Efforts to adopt telephone voting for municipal elections have so far been unsuccessful. In 1993, a group of Colorado citizens placed an initiative mandating that vote-by-phone be used for citizens' initiatives and referenda on the Boulder ballot. The initiative received a 41% "yes" vote, but did not pass. Supporters attributed the failure to misconceptions about the security associated with telephone voting [83].

Electronic Voting Over Computer Networks

Security has long been an important consideration for electronic voting systems implemented over computer networks rather than telephone networks. Much work has been done to develop cryptographic protocols that are both secure and private. In addition, these protocols often possess additional properties, such as complete verifiability, which are not generally obtainable in traditional voting systems.

David Chaum published the first cryptographic voting protocol in a 1981 paper on anonymous electronic mail and digital pseudonyms [28]. This protocol uses public-key cryptography[4] and relies on rosters of digital pseudonyms to conceal the identity of voters. However, the protocol does not guarantee that the identity of voters cannot be traced. Chaum later proposed a protocol that unconditionally conceals the identity of voters [30]. However, elections conducted with this protocol can be disrupted by a single voter. Although Chaum's protocol can detect such disruptions, it cannot recover from them without restarting the entire election [57].

In 1985, Cohen (a.k.a. Benaloh) and Fischer published a description of a secure election scheme in which it is very difficult for dishonest voters to disrupt the election [32]. However, the scheme does not protect the privacy of individuals from the election authority. Cohen later presented extensions to this scheme which distribute the power of government and offer more privacy protection [31]. Benaloh claims this scheme is "reasonably practical" and cites political problems as a greater hindrance to implementation than technical problems. However, he also acknowledges that knowledge of college-level mathematics is required for voters to verify election results independently [12]. In addition, because of the scheme's large communication complexity, casting a vote may take an unacceptably long time [91].

In 1994, Benaloh and Tuinstra proposed a set of verifiable secret-ballot election protocols that do not allow voters to prove the contents of their votes [11]. Unlike the other cryptographic protocols discussed here, these protocols require voters to vote inside a voting booth. The authors maintain that the simplest of their protocols does not require computations on the part of the voter that are outside "the range of normal human ability." However, the more complex protocols that have fewer requirements for trusting election authorities would require the voter to bring a personal computing device into the voting booth. Even

[4] Public-key cryptographic algorithms use a pair of cryptographic keys: a public key and a private key. A message encrypted with a public key can only be decrypted with the corresponding private key and *vice versa.* In addition to protecting secrets, public key cryptography can also serve as a digital signature to authenticate electronic documents.

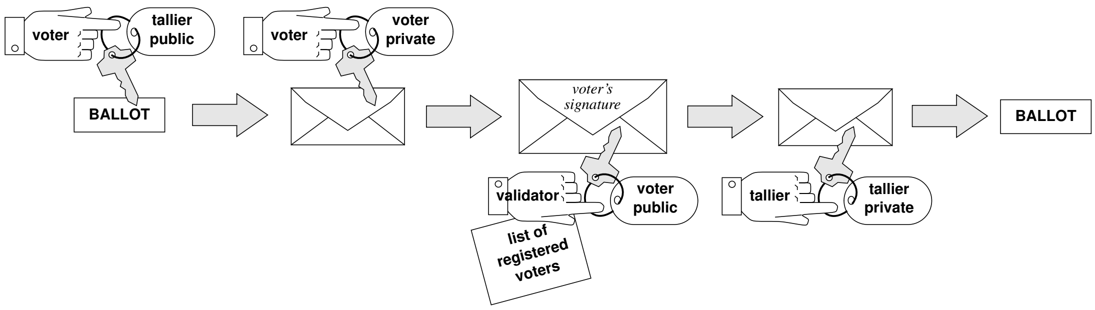

Figure 2.3: A simplistic voting protocol

their most sophisticated protocol does not guarantee that voters cannot be coerced, unless one or more election authorities are trustworthy. Although not a practical solution for Internet voting, the receipt-free nature of this system is significant because it prevents voters from participating in vote buying schemes.

A number of other cryptographic voting schemes have been proposed that require interactions between voters. These schemes, including [37], may be useful in a boardroom setting, but are not suitable for most large-scale elections.

The more practical cryptographic schemes do not require any interaction between voters or use of specialized equipment. However, none of these schemes prevent vote buying. One of the more simplistic of these schemes requires two electronic election authorities: a *validator* and a *tallier*. In this scheme, shown in Figure 2.3, voters encrypt their ballots with the tallier's public key, sign them, and forward them to the validator. The validator strips off the voters' signatures, checks to make sure the ballots were submitted by registered voters who had not yet voted, and forwards the ballots to the tallier. The tallier decrypts the ballots and records the votes. This scheme prevents non-registered voters from voting and registered voters from voting multiple times. However, it only protects voters' privacy if the tallier and validator do not collude. In addition, it does not provide a mechanism for voters to verify that their votes were counted correctly.

In the *Two Agency Protocol* developed by Nurmi, Salomaa, and Santean [80], the responsibilities of validating registered voters and computing and publishing the results of the election are divided between two agencies, as in the simplistic scheme. In this protocol the validator distributes a secret identification tag to each voter just prior to the election. The validator then sends the tallier a list of all identification tags, with no record of the corresponding voters. Each voter sends the tallier his or her identification tag and an encrypted file containing a copy of the tag and the voted ballot. At this point the tallier can make sure the identification tag is valid, but the program has no way of examining the contents of the ballot. The tallier publishes the encrypted file, and the voter responds by sending the tallier the key necessary to decrypt it. When the election is over, the tallier publishes a list of all voted ballots and the corresponding encrypted files. At this point the voters can confirm that their votes were counted properly. Any voter who finds an error can protest by submitting the encrypted file and decryption key again. Because the encrypted file was published earlier, the tallier cannot deny having received it.

The Two Agency Protocol is verifiable by individual voters (unlike the simple protocol discussed earlier), however, it still has several problems. Most importantly, it does not protect voters' privacy if the tallier and validator collude. Thus, the authors state that if the two agencies are going to work together, there might as well be just one agency [92].

The *One Agency Protocol* is identical to the Two Agency Protocol, except for the tag distribution procedure. In the One Agency Protocol, tags are distributed by the tallier (there is no validator) using an ANDOS (all-or-nothing disclosure of secrets) protocol for secret selling of secrets. This solves the collusion problem; however, the ANDOS protocol is quite computationally complex and does not scale well. Another problem with both the One and Two Agency Protocols is that the tallier may cast votes for all voters who have been assigned a tag but do not exercise their right to vote [92, 80].

When Chaum first introduced the concept of blind signatures[5] in 1981 [28], he suggested that blind signatures could be used for secret-ballot elections. Ten years later, Fujioka, Okamoto, and Ohta developed a practical voting scheme that uses blind signatures to solve the collusion problem inherent in protocols like the Two Agency Protocol without significantly increasing the overall complexity of the protocol [48]. (A number of other, less satisfactory blind signature protocols have also been proposed. Sako, for example, proposed a protocol that is simpler but does not completely prevent election administrators from linking ballots with the voters who cast them [90].) The Sensus protocol [35] is based closely on the Fujioka, Okamoto, and Ohta scheme. The main difference between these schemes emerges after the voter has submitted the encrypted ballot to the tallier. In the Sensus protocol, the tallier responds by sending a receipt to the voter. The voter may submit the decryption key immediately after receiving this receipt, completing the entire voting process in one session (verification must still wait until the election is over). In the Fujioka, Okamoto, and

[5]Blind signatures allow a document to be signed without revealing its contents. The effect is similar to placing a document and a sheet of carbon paper inside an envelope. If somebody signs the outside of the envelope, they also sign the document inside. The signature remains attached to the document, even when it is removed from the envelope. Chaum [29] and Schneier [95] provide more background information on public-key cryptography, digital signatures, and blind signatures.

Ohta, protocol the tallier responds by placing the encrypted ballot on a list that is published after all voters vote. Thus, a voter cannot submit his or her decryption key until after the voting phase of the election is over. As a result, votes cannot be cast in a single session.

The Sensus protocol satisfies most of the desirable properties of voting systems well; however, it fails to correct one of the problems inherent in the One and Two Agency protocols: the election administrators (in this case the validator) can cast votes for abstaining voters. These invalid votes can be detected by the abstaining voters themselves or by an auditor who checks the signatures on all the validation requests submitted. However, there is no way to identify the invalid ballots and remove them from the tally. If voters who wish to abstain submit blank ballots, then this problem can be avoided. Another problem with the Sensus protocol is that it requires voters to participate in a complex set of transactions in order to cast a vote. However, the Sensus system employs a *pollster* module that acts as an agent on a voter's behalf, allowing these transactions to be performed with ease. Tests conducted with the Sensus prototype indicate the entire voting process can be completed within a few minutes. While the literature contains many descriptions of cryptographic voting systems, the Sensus system is one of the few cryptographic electronic voting systems to have been built [35]. Another variation of the Fujioka, Okamoto, and Ohta protocol was implemented by Davenport, Newberger, and Woodard [36] and used to conduct student government elections at Princeton University in 1995 and 1996.

Electronic voting has become increasingly popular for use in university elections due to the easy accessibility of computers on college campuses and the relatively low expense of conducting such elections. In 1994, the Association of Graduate Engineering Students at Washington University conducted a referendum election using a simple email voting system. The system resulted in a much higher rate of participation than the organization had previously experienced using traditional voting methods. Electronic voting has also been used successfully at other universities including Dartmouth, Bowling Green, MIT, and Miami of Ohio [5].

Electronic voting has proven controversial on some campuses. At the University of Maryland-Baltimore County, a proposal to conduct student government elections electronically resulted in a heated debate and eventually the withdrawal of the plan. While some members of the campus election board attributed the decision not to proceed with electronic voting to concerns about the security of the system, others maintained that it was politically motivated. The 1995-96 student body president, Lamont King, had been elected with 444 votes in an election in which voter participation was less than 10%. Some students said King feared that the greater voter participation that was likely to accompany an electronic election would require him to put more effort into campaigning outside his own circle of friends and make it more difficult for him to get re-elected. Although the election was eventually conducted via traditional means, King was disqualified after arriving late to a mandatory candidates' meeting, and a write-in slate composed of self-described "computer geeks" won the election [5].

Electronic voting has also become popular on the World Wide Web. Informal Web surveys are easy to implement when security and privacy concerns are ignored (as they usually are). Alexia Parks' Votelink is perhaps the best example of such Web-based voting. The Votelink site invites visitors to vote on a variety of local, national, and international issues. Site maintainers claim that their results are often similar to the results obtained using traditional survey methods. In addition, Votelink has begun working with officials in San Mateo County, California to conduct secure, online polls on issues of local importance [105].

In August 1996, the Reform Party became the first US political party to use Internet and telephone voting to select a Presidential candidate. Voters could also vote in person at the party convention or submit their ballots via postal mail. Regardless of the voting method used, voters' names were affixed to their ballots, eliminating any possibility of a secret ballot. According to a party spokesperson, the party mailed 1.13 million ballots to its members. Voters returned 43,202 paper ballots (via postal mail or at the party convention). Only 3963 voters phoned in their votes and 2101 voters voted via the Internet. The small percentage of voters voting via telephone or computer might be attributable to the fact that the information mailed to voters did not include the toll free phone numbers (the numbers were announced at the convention and in media reports) and included minimal instructions for Internet voting.

Supporters of direct democracy promote electronic voting (via telephone or computer) as a practical solution to the technical problems of polling the entire electorate regularly [83]. Proponents of such schemes include members of the Democratech Party of British Columbia, who advocate governmental decision-making through electronic referenda [22].

When properly designed, electronic voting systems should be suitable for a variety of polling applications, including large-scale elections. Although electronic governmental elections may be months or years in the future, political parties, and professional and social organizations have already started to conduct surveys and elections electronically. While most of these elections currently ignore privacy concerns, advances in email and Web browser software that can easily interface with cryptography software should pave the way for secure and private electronic elections in the near future.

2.3 Motivations for Change

> *…we can conclude that the principles of representation have remained unchanged in most countries throughout the last fifty years. Fundamental changes are rare and arise only in extraordinary historical situations.*
>
> Dieter Nohlen, 1984 [76]

In order to assess the likelihood that a new voting system might be adopted it is useful to understand the circumstances under which organizations are typically motivated to change their voting systems. Major changes in voting systems are rare: at the national level most consistently democratic countries have used the same basic type of voting system for over half a century. Indeed no consistently democratic countries have changed from a plurality system to a PR system or *vice versa* since before World War II [65, 76]. However, smaller changes have occurred at the national level and changes of all types have occurred below the national level and in non-governmental organizations.

Many countries have made minor changes to their voting systems. In Lijphart's analysis of 27 democracies, he found that between 1945 and 1990, 70 voting systems were in use for national legislative elections (lower house only for countries with bicameral systems). Eleven countries had voting systems that remained essentially unchanged during that period; the remaining countries averaged two to three changes per country. In recent years the Greek voting system has undergone the most changes, with five different voting systems in use in Greece between 1974 and 1990. France and Germany each used six different voting systems within the study period, but their voting system changes were spread out over a period of more than 40 years [65].

Major change occurs most often at times of great social and political change. Nohlen notes that the last major period of change for national electoral systems — the early twentieth century introduction of PR systems — "occurred in the context of the democratization of voting rights and of the introduction of universal suffrage; and it was connected with the rise of the labor parties and the restructuring of the traditional party systems…." After major changes occur, minor evolutionary changes may follow as people discover the consequences of the new voting system in practice. Nohlen continues, "The electoral systems in the Western democracies were not invented theoretically, constructed artificially, and then put into practice; to the contrary, most electoral systems were developed historically in a rather long evolutionary process" [76].

In the United States, major changes to local voting systems are often part of a package to overhaul city government. These reforms may be proposed in response to political machines and gerrymandered districts. In the first half of the twentieth century, changes in municipal voting systems were common, due to increased awareness of the significance of the choice of voting systems. Barber explains:

> Electoral system reform came to the fore because representation was at the fulcrum of controversy. Who would be allowed to vote was still a key issue to the excluded — primarily to women and African Americans. There was, however, a growing awareness that how votes were translated into power, the reality of representation, profoundly affected the ability of voters to influence public decisions [6].

Recently, changes to U.S. municipal voting systems began occurring as a result of court enforcement of the Voting Rights Act. In districts that contain sufficiently large, geographically compact, and politically cohesive minority groups that are unable to gain representation, courts have ordered that redistricting occur or that systems of proportional or semiproportional representation be instituted [6].

Change may also occur when people perceive a tangible "problem" with the current voting system. Often the problem is based on a popular belief that the results of a past election or the predicted results of an upcoming election are unfair or even perverse. Sometimes a voting system will be changed to maintain the power of a ruling majority that is slowly losing its support. By changing the voting system the ruling majority can thus prevent the minority from attaining the votes needed to gain power. In municipal governmental voting in the United States, such a change has often been racially motivated [6].

Change occasionally occurs because people believe a new voting system is inherently superior to the current system or because they are curious about a new system. This generally occurs only in professional societies.

Generally, political circumstances motivate change only in the types of vote aggregation procedures used or the distribution of voting districts. However, as explained in Section 2.2.4, such circumstances may motivate change in the vote-collection system used, especially when such a change is likely to have a large impact on voter turnout.

In the remainder of this section, several historical cases in which organizations or governments changed voting systems are presented and discussed.

2.3.1 Proportional Representation in Ireland and the United States

Although theorists tend to favor the single transferable vote system to list PR, list PR is the most widely used system among Western democracies [66]. Ireland, Malta, and

Australia are the only democracies that currently use STV at the national level [65]. Over two dozen U.S. municipalities have used STV at one time or another, but only the city of Cambridge, Massachusetts still uses this system.

In 1911, STV was first proposed in Ireland by the British Proportional Representation Society as a means to prevent influential minorities from forming their own governments rather than submitting "to the jurisdiction of an all-Ireland parliament." Following a public lecture on STV, the Proportional Representation Society of Ireland was formed. In 1912, the Irish Home Rule Bill passed with amendments adopting STV for some constituencies to the new Irish House of Commons and, after five years, all seats in the Senate. But "the outbreak of war in August 1914 and the immediate deferment of the Home Rule Act meant that the new system was never put into operation" [82].

In 1918, STV was adopted in Sligo, a small borough in western Ireland. Unlike the general election of 1918, which was characterized as "bitter and ugly," Sligo's first STV election resulted in what was generally accepted by all parties as "fair" representation. In 1919, STV was adopted for local elections throughout Ireland. Amidst great political turmoil, guerrilla warfare, and disagreement about whether Ireland should remain a part of the United Kingdom, the Government of Ireland Act of 1920 established two Irish political systems. Each government was to have a bicameral parliament, the lower house of which was to be elected with STV. The 1921 parliamentary elections resulted in remarkably unproportional representation due in part to the fact that all the constituencies in the south were uncontested [82].

In December 1921, representatives of the Irish and British governments signed a treaty establishing the Irish Free State and giving the Irish counties dominion status. Northern Ireland opted out of the treaty in May 1922. In June 1922, a draft constitution was proposed for the Irish Free State. This constitution provided for a British-style system of government, but called for the legislature to be elected "upon the principles of Proportional Representation." O'Leary attributes the decision to adopt PR to three factors: the fact that PR had been recently adopted in other European democracies, continued campaigning by the PR Society, and "the necessity for satisfying the fears and susceptibilities of a powerful minority, the southern Protestants." Although the draft constitution did not indicate what type of PR system to use, it was interpreted as specifying STV rather than list PR, as STV was the only PR system familiar to most of the Irish representatives. The Constitution came into effect in December 1922. A subsequent constitution adopted in 1938 specified STV explicitly [82].

Since adopting STV, Ireland has had a fairly stable government, a high degree of proportionality in representation, and a high rate of voter participation. Despite the complicated nature of this voting procedure, voters rejected proposed constitutional amendments to replace it with a plurality system in 1959 and 1968 [50, 106].

STV has proven controversial when adopted by American cities. Cincinnati, Ohio adopted the system in 1924 in response to severe gerrymandering of city council districts. Voters rejected referenda to repeal the system in 1936, 1939, 1947, and 1954, but accepted a repeal referendum in 1957 that replaced STV with an at-large plurality election system. It is generally believed that the referendum succeeded due to threats that the STV system would lead to the election of the city's first black mayor. Referendums to reinstate STV failed in 1988 and 1991 [6, 43].

After the 1991 defeat of STV, African-American voters took their case to court. Barber provides the following account:

> …fifteen African American voters sued the city in federal court, alleging that the plurality/at-large electoral system for city council was adopted with the intent to discriminate against black voters, and that it indeed caused minority vote dilution in violation of the Voting Rights Act. The remedy they sought was …a single-member district system. Judge Herman Weber …delayed the trial to allow the city council itself to present its preferred remedy to the voters.… [T]he council placed on the May 1993 ballot a council election plan retaining the nine at-large members but providing for their election by cumulative voting. A brief but intense campaign led to defeat of the initiative by a 79% majority, and the case went to trial.…
>
> In July 1993 Judge Weber issued his decision upholding the city's 9X voting system. Although conceding that a nasty racial "whisper campaign" had accompanied the 1957 repeal of PR and the adoption of plurality/at-large voting, the Judge denied that city officials were responsible for it [6].

Promoted by the Proportional Representation League from 1893 to 1932 and later by the National Municipal League, PR has failed to replace plurality voting for state or national elections in the United States. After PR supporters began focusing on local elections, PR systems were adopted by about two dozen American cities during the first half of the twentieth century. As with the Cincinnati, most of these cities adopted PR to combat gerrymandering and party machines. Cities generally adopted PR through referendum efforts led by PR League and often supported by women's organizations. However, state party leaders opposed PR and fought it in the courts and state legislatures. Some state legislatures voted to outlaw PR in all municipalities. In Michigan and California, judges with close ties to party leaders ruled that PR elections violated state laws. But courts in Ohio and New York refused to find PR in violation of state law. Eventually referenda were introduced to repeal PR in the cities where it remained in use. These referenda ultimately succeeded in every US city except Cambridge. The success of these referenda has been attributed to a changing political climate and the fact that counting STV ballots by hand is an extremely time-consuming process. Barber proposes that another "explanation for repeal seldom recognized in the literature is the fact that PR/STV did what it was supposed to do, that is, facilitate the representation of minorities of various sorts" [6].

The only other public governing bodies in the United States that currently use the STV system are the 32 community school boards in the City of New York [43]. However, the system is used by other institutions, including the Faculty Senate at Washington University.

2.3.2 Approval Voting in Professional Organizations

There are also several interesting cases involving organizations that switched to approval voting in the 1980s. The Mathematical Association of America, the American Statistical Association, and The Institute of Management Sciences adopted approval voting primarily out of scientific curiosity [21]. ASA and TIMS conducted experimental elections prior to adopting approval voting [19].

The Institute of Electrical and Electronics Engineers, a professional organization with over 300,000 members, adopted approval voting in 1988 in order to prevent the election of a presidential candidate "who was frequently critical of IEEE and its policies" [21]. This candidate had come within 242 votes of beating both the Board-nominated candidates in the 1986 election. Irrespective of their motivations in 1988, approval voting was probably a sensible choice for the IEEE Board to select if they wished to continue their practice of nominating two candidates for each position. Many other professional organizations nominate only one candidate for each position. In the absence of self-nominations, all positions are uncontested and there is little interest in the election. By nominating two candidates, the IEEE Board can be assured of interest in the election. However, as the two Board-nominated candidates tend to have similar ideas, they may face difficulties when running against self-nominated candidates who often have quite different ideas about how the organization should be run. Under a plurality voting rule, a self-nominated candidate with only minority support may win the election if the majority splits their votes between the Board-nominated candidates. By adopting approval voting the IEEE Board was assured that their candidates could lose only to a self-nominated candidate preferred by the majority.

The first political organization to use approval voting was the Pennsylvania Democratic party, which used approval voting for a non-binding Presidential preference survey in 1983 [75]. As part of this survey, the Pennsylvania leaders were questioned about their attitudes towards approval voting. Two-thirds of the 201 delegates reported that they were sufficiently informed about approval voting to form an opinion about it. Nearly three-fifths of these delegates reported that they had formed favorable opinions.

2.3.3 Academy Awards Documentary Nominating Committee

The documentary nominating committee of the Academy of Motion Picture Arts and Sciences switched from one cardinal utility voting method to another in 1995 after a popular documentary film failed to be nominated. The nomination vote is a key part of the process by which the Academy award winner is selected.

To select its five nominations, the documentary nominating committee first screens all eligible films, turning off the projector before the film is over if more than half the committee members dislike the film. If a film is not screened in its entirety it is eliminated from further consideration. The committee members each rate the remaining films on a scale of 4 to 10. The top five films receive nominations. After the nominations have been made, any Academy member who screens all five films is eligible to vote to select the winner [2].

In 1995, some of the documentary committee members reportedly lobbied their colleagues to give the movie *Hoop Dreams* the lowest score possible so as to prevent its nomination. They argued that, due to its popularity, if *Hoop Dreams* was nominated it would likely win, regardless of the other documentaries nominated. At least three of the committee members are believed to have given *Hoop Dreams* a 4 on this account. There are 47 members of the committee, but the Academy directors would not say how many actually voted [2].

After *Hoop Dreams* failed to win a nomination, the Academy directors decided to change the voting procedure used in future years. Starting in 1996, the movies will be rated on a scale of 6 to 10 [2]. This change will make it more difficult for a small number of Academy members to manipulate the nomination vote; however, it will not eliminate the possibility of such manipulation.

2.4 Summary

This chapter has provided a general overview of the types of voting systems available and the issues that must be considered when selecting a voting system. We have seen that no perfect vote aggregation procedures exist for choosing between three or more alternatives. We have also seen that advances in computer technology have made new types of ballot collection systems possible and made complicated vote-aggregation procedures more feasible. Finally we have seen that major changes in voting systems at the national level are rare, but that minor changes at the national level and major changes at the local level and in non-governmental organizations are more common. Changes in voting systems most often occur at times of great political change and in response to perceived problems with the results of a past election or the expected results of a future election. Voter education is usually necessary to convince people to adopt a new system.

In the next chapter several of the issues presented here will be discussed in more detail and the motivations and design for a new vote aggregation procedure will be presented.

Chapter 3

Declared-Strategy Voting Systems: Motivations and Design

> *The method of choice...should be as scientifically framed and as efficiently adapted to its purpose as experience and political wisdom can make it.... It is not sufficient for every citizen to have the vote; he should also be assured of the greatest possible freedom and effectiveness in its use.*
>
> Enid Lakeman and James D. Lambert, 1955 [64]

Declared-strategy voting (DSV) is a new voting paradigm that does not fit neatly into the categories described in Chapter 2. Like the utilitarian systems, DSV — when outfitted with the rational strategy formulator to be introduced in Chapter 4 — requires voters to supply cardinal preference information; however, this information is not summed or multiplied to determine the election outcome. Like the positional systems, DSV uses ordinal preference information; however, DSV also considers a voter's relative strength of approval for each alternative. And there are many types of DSV systems, some of which are single stage and some of which are multi stage. Thus, DSV is quite a departure from the voting systems previously investigated.

What do we hope to gain by departing from the traditional approach to voting system design and where does this departure lead us? In this chapter we explain the motivations behind our approach and present some specific instances of declared-strategy voting systems.

3.1 Motivations

This section explores the major motivations behind the new declared-strategy voting paradigm in detail. Three ideas motivate declared-strategy voting: the possibility of taking advantage of electronic voting systems to achieve more beneficial methods of vote aggregation than are feasible with traditional systems, the desire to find an information-neutral voting system, and the desire to maximize a voting system's expressiveness.

3.1.1 Taking Advantage of the Electronic Medium

With the advent of accessible and affordable high-speed computers and interconnection networks, elections and surveys are starting to move from the voting booth, postal mail, and telephone into the computer. Voting over computer networks is likely to appeal to geographically distributed organizations such as professional societies. Existing methods of conducting surveys and elections electronically are focused on emulating (to the greatest extent possible) the feel, look, and mechanics of traditional surveys and elections, while taking advantage of the speed, accuracy, and computational power offered by the electronic system. Such approaches are quite reasonably founded on placing the computer and its associated technology in the background, so that participants suffer little shock in moving to electronic means of expressing choice. These methods thus improve on traditional voting systems, by allowing voters to cast secret ballots without being physically present at an official polling place and by introducing new ways to verify the accuracy of the election results [79]. However, these systems do nothing to improve the vote aggregation procedure itself.

A goal of this research has been to explore a new method of expressing choice that is only now made feasible by advances in computing and communications technology. Rather than simply transferring extant paradigms to the computer, this dissertation investigates the possibility of using computers to conduct surveys and elections in more beneficial and meaningful ways.

3.1.2 Information Neutrality

Voting theorists have long been aware that it is not always in the best interest of voters to vote for their most sincerely preferred candidates. Voting schemes in which voters can obtain a more preferred outcome by voting *strategically* rather than *sincerely* are dubbed *manipulable*. And it has been well established that most voting schemes used in elections with three or more alternatives (including all reasonably "attractive" voting schemes previously investigated) are manipulable [7, 52, 54, 94]. On the other hand, there are well-known systems that rely on randomness to elicit sincere voting behavior [7]; however, the arbitrariness present in such systems renders them generally unacceptable to voting populations.

Studies have shown empirical evidence that voting systems are manipulated by voters in real elections. Brams and Merrill [20] analyzed data collected in the 1992 National Election Study and estimated that 34.5% of Perot supporters did not vote sincerely in the 1992 Presidential election because they felt they could make better use of their votes by voting for their second-choice candidates. Cain [24] developed a model of strategic voting in the British electorate that suggests strategic manipulation does occur, especially when voters want to avoid "wasting" their votes or when the race is only close between two of the parties in a three party election. And Black [14] analyzed data from the 1968 and 1972 Canadian Federal elections and estimated that 12% of the voters in these elections voted for their second-choice party. It is difficult to determine how many of the voters participating in the analyzed elections voted sincerely even though they could have benefited from voting strategically. And it is uncertain whether these sincere voters voted sincerely out of ignorance of the preferences of others, ignorance of how to use preference information to their best advantage, a desire to show support for an underdog candidate, or a belief that insincere voting is dishonest.

Indeed, early criticism of manipulable voting systems equated manipulation with dishonesty. The eighteenth century mathematician Jean-Charles de Borda is said to have responded to criticism that his Borda count voting scheme was manipulable by saying, "My scheme is only intended for honest men" [13]. But later theoreticians dismissed Borda's assumption that people are honor-bound to vote for their most preferred candidate. In an introduction to approval voting published in 1983, Brams and Fishburn wrote, "If some voters can do better by strategically voting for candidates other than those they most prefer, then, instead of accusing them of dishonesty in making rational strategy choices, it seems better to try to design systems that make dishonesty unnecessary or costly in order for voters to achieve their aim of electing desired candidates" [17]. Earlier, in an 1876 pamphlet[1] written well before the development of game theory, C. L. Dodgson had suggested that an election be thought of "more as a game of skill than a real test of the wishes of the electors." He proposed that "as my own opinion is that it is better for elections to be decided according to the wish of the majority than of those who happen to have most skill in the game, I think it desirable that all should know the rule by which this game may be won." Although Dodgson elaborated on the rules for winning certain types of games, little additional work was done in this area until 1953 when Farquharson set out to apply game theory to voting procedures [44].

[1] Dodgson's unpublished pamphlet "A method of taking votes on more than two issues," is reproduced as an appendix to [13].

Farquharson began by introducing the concept of a *voting strategy.* Farquharson described a voting strategy as a plan made prior to an election that prescribes the course of action a voter should take given any contingency that can arise. In a single round election, a strategy is simply the voter's plan for how to cast a single ballot. However, in a multiple round election such as one that involves choosing between two alternatives at a time, a strategy must include a plan for all pairs of alternatives that could possibly be presented. Farquharson also discussed *sincere* voting, in which voters always vote for their most preferred alternatives, and *sophisticated* voting in which voters select utility maximizing strategies that take into account the strategies of the other voters [44].

The terms sincere, strategic, and sophisticated voting have varying definitions in the literature. Throughout this text we use the following definitions.

Sincere vote: A vote such that for any two alternatives under consideration, the voter has given greater or equal weight to the more preferred of those alternatives.

Strategic vote: A vote that is rational, but not sincere. Often such a vote will be determined through decision-theoretic analysis, which takes into account the preferences of the other voters. One type of strategic vote is a **sophisticated vote**, a vote determined to be optimal through game theoretic analysis, which takes into account the likely strategies of the other voters. Sophisticated votes may be viewed as higher-order functions than the simple strategic votes we consider in this dissertation.

Strategic or sophisticated votes shall also be referred to as *insincere votes*.

The application of game theory to voting theory brought on a variety of new approaches to studying voting schemes and voter behavior. It led to further investigation of the conditions under which voting schemes might be immune to manipulation, and it led to the development of models designed to provide rational choice explanations for voter behavior.

Several voting theorists, including Farquharson and Dummett [41], conjectured that it would be impossible to find a voting scheme immune to manipulation. Indeed, in the early 1970s, Gibbard and Satterthwaite independently proved that all nondictatorial voting schemes that have at least three possible outcomes, that lead to a single choice, and that do not rely on chance, are manipulable [54, 94]. Gibbard also demonstrated by example the existence of a "mixed decision scheme" that is neither manipulable nor dictatorial, and can allow more than two outcomes. In this scheme each voter submits a ballot containing a vote for a single alternative. One ballot is then selected at random, and the alternative specified in that ballot is declared the winner. This scheme is unsuitable for most purposes because it relies heavily on chance. Gibbard suggested that further work be done to explore decision schemes that do not leave "too much to chance." This suggestion was taken up by Barbera [7], who showed that "the only selection methods that can be both nondictatorial and nonmanipulable are those in which chance plays an extensive role." Gardenfors [52] also extended the work of Gibbard and Satterthwaite, focusing on voting schemes that do not necessarily select a single outcome. He showed that most of these schemes are manipulable, and those which are not tend to be very indecisive.

While these theorists were exploring the extent to which voting schemes are manipulable, other voting theorists sought to determine the optimal strategies for voters to use. McKelvey and Ordeshook developed "A General Theory of the Calculus of Voting" in which they derived decision rules that could be used by voters to determine their optimal strategies [69]. This article extended "A Theory of the Calculus of Voting" in which Riker and Ordeshook derived decision rules that could be used to determine whether it was rational for a particular voter to vote at all [87]. The theory was further generalized by Hoffman in "A Model For Strategic Voting" [55].

All of the decision models cited here are expected-utility models that require voters to consider their personal preferences as well as the probable preferences of the rest of the electorate. Information about the preferences of others allows voters to determine the relative probability of each candidate winning the election. When voters are able to determine such probabilities, they are said to be making decisions under *risk.* Without information about the preferences of others, voters are not able to determine the probabilities of the various contingencies. Merrill showed that when this occurs the sincere strategy is always the optimal strategy in plurality, Borda, and approval voting elections [70]. This is likely the case for other voting schemes as well. Merrill's results suggest that voters are only able to manipulate voting systems when they are making decisions under risk: that is, when they know the probability distribution for the various election outcomes. When voters have no information about the preferences of others they cannot manipulate the voting system.

Others have also pointed out that the fact that a voting system is manipulable does not imply that it will actually be manipulated. As Gibbard explained:

> …to call a voting scheme manipulable is not to say that, given the actual circumstances, someone really is in a position to manipulate it. It is merely to say that, given some possible circumstances, someone could manipulate it [54].

We assert that the circumstances required for manipulation are:

1. a set of preferences for each voter such that at least one voter or group of voters can obtain a more preferred outcome by voting insincerely,

2. at least one voter who has sufficient information about the preferences of other voters to determine that he or she can obtain a more preferred outcome by voting insincerely, and

3. at least one voter who has sufficient knowledge about how to solve utility-maximization problems to determine his or her utility-maximizing strategy.

In an election where the required circumstances for manipulation are present, not all voters who could benefit from voting strategically will have sufficient knowledge and information to formulate a utility maximizing strategy. As we have discussed, the ability to manipulate is dependent on the individual voter's ability to gather information about the preferences of the other voters, but there may be costs associated with gathering this information [14]. Depending on the type of election, money, education, access to media, time, and political alliances may be necessary resources for gaining sufficiently accurate voter preference information. Because of the difficulty some voters may have in obtaining this information, it is conceivable that voters might accept inaccurate estimates — perhaps propagated by certain candidates or their supporters — as the truth. Indeed, it has been reported that supporters of Pat Buchanan actively sought out poll takers during the 1992 and 1996 Republican Presidential primaries so that pre-election polls would show more support for Buchanan than actually existed [26]. In addition, as noted by Black [14], some voters "may hold probability estimates that are inflated or clearly erroneous or the product of the simple mimicking of the views of a close friend or spouse." Moreover, the ways that poll data and other preference information flow through society and are interpreted by voters are not well understood [16].

Thus, we conclude that a significant problem with manipulable voting systems is that they allow voters with more accurate information about the preferences of others to use their votes more effectively than other voters, essentially granting these voters a weighted vote. This is contrary to the principles espoused by many democratic countries and organizations in which all people are given the same amount of say in the electoral process, regardless of their means. Those who subscribe to these principles often expect that by granting each individual one vote, they are granting all people equal voting power. However, this is not necessarily the case when some voters have the means to obtain information about the preferences of other voters that is unavailable to everyone. These voters may use this information to formulate optimal voting strategies which cannot be identified in the absence of such information.

Another problem with manipulable voting systems is that not all voters understand the formulation of utility-maximizing functions. Thus, even if all voters have equal information about the preferences of others, those who understand the formulation procedure have more power than those who do not. Riker [86] notes that when strategic voting occurs frequently, the meaning of social choices "may consist simply of the tastes of some people (whether majority or not) who are skillful or lucky manipulators."

A goal of this research has been to design a voting system that approaches information neutrality: that is, a system in which all voters will have equal access to both the information and knowledge needed to vote strategically.

3.1.3 Expressiveness

Manipulable voting systems frustrate attempts at analyzing and interpreting election results because analysts cannot distinguish between voters who voted sincerely and those who voted insincerely. Thus, election results give little indication as to whether a winning alternative had strong support from those who voted for that alternative. As noted by Knight and Johnson [62], "Social choice theorists demonstrate that any electoral outcome is at least partly an artifact of the aggregation mechanism through which it is produced. Therefore, electoral results always require interpretation and justification." Insincere voters add a degree of uncertainty to such interpretation.

In addition, voters correctly view elections as opportunities to voice their opinions to their government. They may resent the fact that in order to get the most value from their vote (voting strategically) they in effect lose their opportunity to make their true opinions known. These voters basically have to choose between casting an effective or an expressive vote.

Brams and Fishburn [17] suggest that if voting systems allowed voters better opportunities to express their preferences, more people would be likely to vote:

> Voters who think they might be wasting their votes, or who cannot decide which of several candidates best reflects their views, would not be on the horns of a dilemma. By not being forced to make a single — perhaps arbitrary — choice, they would feel that the election system allows them to be more honest. We believe this would make voting more meaningful and encourage greater participation in elections.

Thus, a goal of this research has been to develop a voting system that will allow voters to vote effectively while still expressing their opinions. Such a system would give voters the satisfaction of expressing themselves and analysts the sincere data with which to interpret election results.

3.2 Declared-Strategy Voting System Design

If there were a way to elicit sincere preference information from all voters prior to an election, this information could be made public and all voters would have the opportunity to use it in formulating their strategies. In the absence of sincere preference information, information about the strategies the other voters plan to use (sincere or otherwise) is still useful. However once the information is published, voters will adjust their strategies to take into account the preferences of others. Myerson and Weber [74] describe a series of polls in which voters are given the results after each poll. Eventually, they predict, a *voting equilibrium* may arise in which "the perceptions arising from the publication of the poll lead the voters to behave in a manner that in turn justifies the predictions of the poll." They then go on to prove that at least one voting equilibrium must exist in all elections. In this scenario, all voters have equal access to the information that they need to vote strategically. However, conducting an election in this manner is unattractive because it would require voters to go repeatedly to the polls. This sort of election is likely to take a long time and have low voter turnout.

One way to side-step the problem of supplying voters with equal access to information is to allow them to cast votes that are contingent on the votes of others. While some parliamentary voting rules allow absent voters to cast proxies that are contingent on the votes of others (for example, the system used by the French National Assembly [44]), contingency voting is not a feature of most voting schemes. Contingency voting has a fundamental problem in that non-contingent votes must be counted before contingent votes; if all voters wish to cast contingent votes there is no way to count any votes without introducing an element of chance to decide which votes to count first. Another problem with contingency voting is the difficulty in evaluating complex contingency rules, especially when the number of voters is large or when votes are tallied at many geographically distributed precincts. But contingency voting is nonetheless appealing because it allows voters without voter preference information to vote strategically. In addition, examination of contingency votes may provide insight into the true preferences of the electorate that is not otherwise available when voters vote insincerely.

The advent of high-speed computers and computer networks makes it feasible to tally contingency ballots in a reasonable amount of time. In addition, computers can handle complicated vote aggregation methods, making possible voting procedures not previously considered. With that in mind, we now sketch the design of an information-neutral voting system that employs contingency voting. We call this system a *declared-strategy* system because voters declare the computations used to determine their vote. The computation represents the voter's strategy in formulating a vote, rather than simply the outcome of that decision process. Following game-theoretic practice, we assume that each voter decides on a *strategy* prior to the election that includes a specification of how the vote will be cast given any contingency. A declared strategy is a first-order function of the "state" of the election. For example, in the 1992 US Presidential election, a Perot supporter who preferred Clinton to Bush might have used the strategy: *vote for Perot if he has received more votes than Clinton so far, otherwise vote for Clinton.*

In our system, voters submit their strategies to the election computer. The computer evaluates the voters' strategies using the current election tally and other state information. The strategies are then aggregated to determine the election outcome. Thus, our basic system requires a *strategy aggregator* module that includes a *strategy evaluator* for evaluating each strategy and a *vote accumulator* for tallying the results. However, there are many ways in which we might proceed to flesh out the details of our DSV strategy aggregator. We developed two types of approaches, which could be considered as the extreme ends of a spectrum of approaches: batch aggregation and ballot-by-ballot aggregation. Both of these techniques may be combined with any positional aggregation technique (plurality, approval, Borda, etc.). We describe these two approaches below.

3.2.1 Ballot-by-ballot Aggregation

The ballot-by-ballot technique involves making an outcome prediction, evaluating one selected ballot based on that prediction, updating the prediction, and repeating until all ballots have been evaluated. The initial prediction may be set to zero or based on some initialization procedure. As the ballots are evaluated, they are added to a running tally. The first ballot selected will be evaluated with no information (unless the system is initialized with some information) while the last ballot selected will be evaluated with perfect information. The strategy aggregator needed to conduct an election using this technique can be represented by the machine model shown in Figure 3.1. This technique results in outcomes that may be dependent on the order in which the ballots are selected for evaluation. As a result, the election results may not be stable. Note that the traditional voting systems discussed in Chapter 2 always produce the same outcome given the same set of ballots (with the

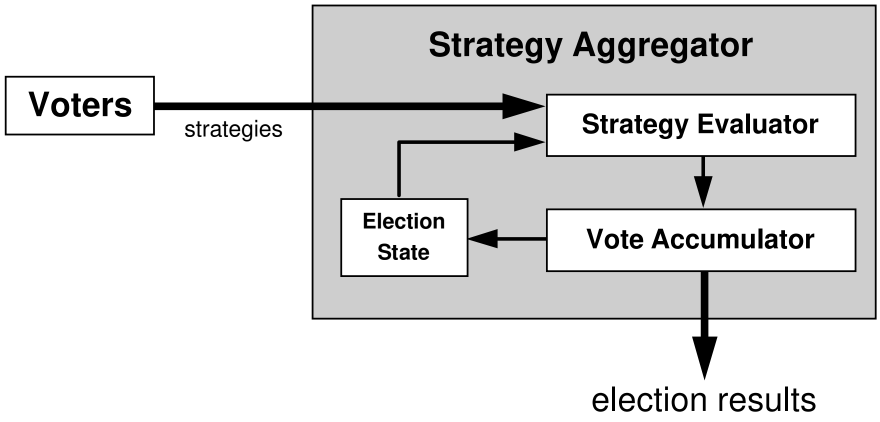

Figure 3.1: A machine model of ballot-by-ballot declared-strategy voting

exception of some PR implementations that employ randomness), and thus are completely stable. Gibbard's mixed decision scheme [54] is an example of a highly unstable voting system, as the outcome is based almost entirely on chance. DSV systems may be designed to minimize instability (or even eliminate it at the expense of increased manipulability).

Any ordering might be selected for ballot evaluation; however, a random ordering is perhaps the most fair, while considerably reducing the opportunities for the system to be manipulated. Although the results may be unstable, there are voting situations in which this ramification is acceptable. For example, John M. Brimacombe, managing director for a small British software company, found an unstable voting system to be the best solution to a voting problem faced by his employees four years ago. Brimacombe explained that, due to limited resources, he decided to reduce the recreational USENET newsgroups that were offered on his company's server from approximately 300 groups to about 100 groups. He concocted a cumulative voting scheme in which each employee could distribute 100 votes over his or her favorite newsgroups. Each group was then given a weight based on its average traffic volume and the groups were ranked according to their vote totals divided by their weights. He then went down the list, selecting the top ranked newsgroups in order until the sum of the average traffic of the selected groups equaled the limit that he had set for the company. When he announced the results to his five employees, all were dissatisfied. Some groups had received many surplus votes at the expense of others that just barely missed the cutoff. The employees asked to repeat the voting procedure with their newly acquired knowledge about the preferences of their colleagues. However, the second vote proved to be not much better than the first, and the voting was repeated several more times. It is likely that had the voting continued indefinitely, an equilibrium would have developed eventually. But in the interest of time, Brimacombe decided to choose an alternate voting system. This time he selected one employee at random and had him announce his votes. He then selected another employee to vote after observing the votes of the first employee. This procedure continued until everyone had cast their votes. All employees were satisfied with the outcome, despite the fact that it was likely to have been slightly different had the employees voted in a different order [59].

The procedure Brimacombe ended up using is similar to ballot-by-ballot DSV. However, Brimacombe's method would not scale well if more voters were to be polled because it requires voting to take place serially. Ballot-by-ballot DSV improves on this procedure by allowing each voter to submit a strategy prior to the election that represents the voter no matter when it is selected for evaluation. Thus voters need not wait to see how their peers voted before casting their ballots.

3.2.2 Batch Aggregation

The batch technique involves making an outcome prediction based on the sincere outcome point, evaluating all ballots based on this prediction, updating the prediction based on the previous evaluations, and repeating until an equilibrium is reached. This technique simulates the situation described by Myerson and Weber [74] in which voters are repeatedly polled until the results of a poll justify the predictions of the previous poll. However, batch DSV requires that the voters be polled only once.

The batch technique may be preferable to the ballot-by-ballot technique in some situations because it need not introduce chance into the declared-strategy system. Although at least one equilibrium always exists [74], batch DSV may not reach an equilibrium under all circumstances and therefore requires the use of an arbitrary termination condition. A machine model of batch DSV is shown in Figure 3.2.

The idea of using multiple batched rounds to provide decision-makers with increasing amounts of information has been applied in a variety of applications, most notably the *simultaneous ascending auction* protocol used by the Federal Communications Commission (FCC) for recent electromagnetic spectrum auctions. This auction method was proposed by economists Paul Milgrom, Robert Wilson, and Preston McAfee. It allows multiple licenses to be bid on simultaneously in a multi-round process. The results of each round are

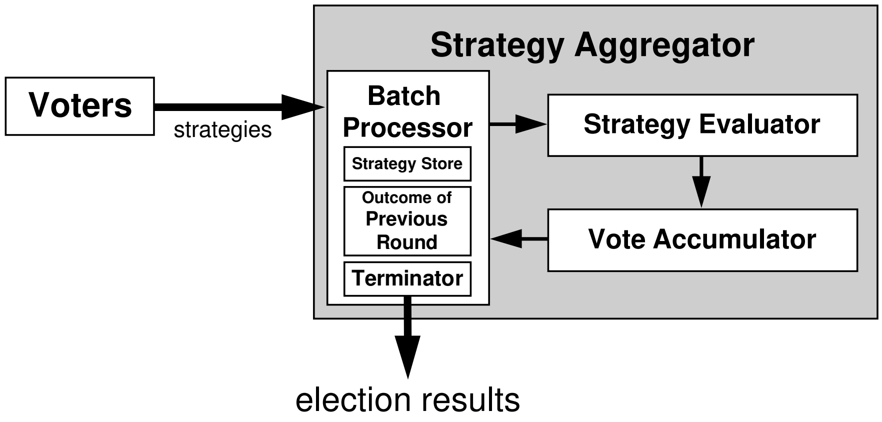

Figure 3.2: A machine model of batch declared-strategy voting

announced prior to the start of the next round, thus providing bidders with increasing information about their competitors' strategies as the auction progresses. Bidding proceeds until an equilibrium is reached or a sufficient number of rounds have transpired that the auction is terminated. The FCC conducted their auctions in three stages, each with different conditions on the minimum activity of each bidder. This forced bidders to participate and reveal information in every round rather than waiting to observe their competitors' strategies before bidding. The auction was conducted online to facilitate the lengthy bidding process. The use of the simultaneous ascending procedure resulted in companies being able to win groups of complementary spectrum allocations (which is often difficult when auctions occur serially) and in the U.S. government receiving payments much higher than expected [68].

Batch DSV operates in a similar manner to the simultaneous ascending auction. However, formulating an optimal voting strategy is a somewhat simpler problem than formulating an optimal bidding strategy, especially when the election involves independent alternatives. DSV thus allows voters to submit a strategy that will represent them throughout the election rather than resubmitting their ballots after each round. In the next chapter we present an automated, rational strategy formulator that formulates optimal voting strategies on behalf of voters.

Chapter 4

Rational Strategy Formulation

The overall design and operation of declared-strategy voting systems was introduced in the previous chapter. We explained that DSV systems are designed to allow voters to express their preferences in terms of a strategy that results in a vote that is both expressive and effective. Throughout the chapter the existence of such voting strategies was assumed, but no attention was paid to their form or how they might be determined. In this chapter voting strategies will be explored in detail and a method for rational strategy formulation will be presented.

Because rational strategy formulation is a complicated process, it is crucial that an automated strategy formulation system be provided so that voters need not study decision theory in order to cast a vote. Here we develop a DSV *strategy formulator* that can process information supplied by a voter and produce a rationally-optimal strategy. Figure 4.1 illustrates how the strategy formulator fits into the model developed in Chapter 3.

Before formalizing the notion of voting strategies, we examine the types of strategies voters might use (consciously or subconsciously) in a traditional election with three or more alternatives.

Some voters may adopt a *sincere* strategy, refusing to vote for any alternative besides their favorite, regardless of the favorite's chances of winning.

Other voters may exhibit more strategic behavior, taking into account both their own preferences and the likely preferences of other voters. For example, these voters may adopt a *don't waste my vote* strategy in which they vote for their sincerely preferred alternative only if that alternative is likely to be one of the top contenders. A variation on this is the *vote against last choice* strategy in which voters vote for whichever alternative has the best chance of beating their least favorite alternative.

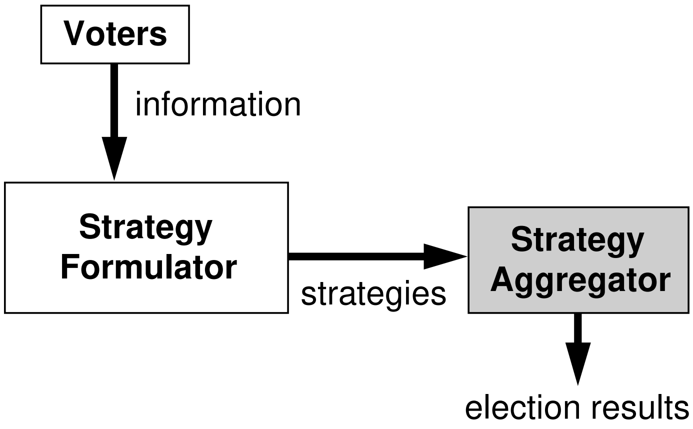

Figure 4.1: Declared-strategy voting with strategy formulator

Some voters may be fairly indifferent between the alternatives and may select strategies based on factors not related to their preferences for the various alternatives:

- Voters who enjoy supporting the winning side or who want to show strong support for the selected alternative regardless of outcome may adopt a *bandwagon* strategy in which they vote for the alternative most likely to win.

- Those who wish to show that the winner is not strongly supported or express sympathy for a candidate who is unlikely to win may adopt an *underdog* strategy in which they vote for the alternative mostly likely to come in last place.

All of the above strategies may be considered rational (assuming they are consistent with a voter's preferences) in that they further the outcome most desired by the voter. Throughout this text we assume that *voters are rational if they take the actions they believe are most likely to lead to their most preferred outcome — regardless of whether it is reasonable for them to prefer that outcome.* Thus, in order to behave rationally, a voter must have a set of preferences, a set of beliefs, and a strategy consistent with his or her preferences and beliefs.

While it is reasonable that voters should seek to vote rationally — and thus possibly insincerely — it has not heretofore been possible for the voter to express a rational voting strategy in a traditional election. DSV provides the vehicle for expressing a rational strategy; however, the mathematics associated with formulating the optimal rational strategy can be formidable, certainly beyond the capability of a mathematically unsophisticated, albeit politically astute voter.

In this chapter we develop an automated strategy formulator that produces strategies which optimally represent the rational voter in a DSV election. Section 4.1 introduces a formal method for determining a voter's rationally optimal strategy. Section 4.2 discusses methods for determining and quantifying beliefs about the preferences of other voters and introduces a novel method that scales better than those found in the literature. Section 4.3 examines the specification of strategies for voters who are indifferent between the alternatives. These elements are tied together and a decision-theoretic strategy formulator is presented in Section 4.4.

4.1 Expected-Utility Model

Several authors have presented evidence that some voters use expected-utility models involving their personal preferences and the probable preferences of the rest of the electorate to determine their rationally optimal strategies in actual elections. In fact, Black [14] used the approach outlined by McKelvey and Ordeshook [69] to model the 1968 and 1972 Canadian Federal elections and found "significant empirical support for the role of probabilities in the multicandidate calculus of voting."

McKelvey and Ordeshook [69] deduced that in N-candidate elections one can establish N difference equations that model the *expected gain* in utility associated with a given voter voting for a particular candidate rather than abstaining. Assuming that the voter has established a set of utility values $(u_1, u_2, \ldots u_N)$ specifying his or her personal utility for each candidate winning the election, and assuming that the probability of a three-way tie is negligible, the expected-gain equations for a plurality election that selects a single winner can be written in the form

$$E_i - E_0 = \sum_{i \neq j}^{N} p_{ij}(u_i - u_j) \tag{4.1}$$

where E_i is the expected utility associated with voting for candidate i, E_0 is the expected utility associated with abstaining, and p_{ij} (which we shall refer to as a *pivot probability*) is the probability that the voter will be *decisive* in creating or breaking a first-place tie between i and j. For elections that select M alternatives, the pivot probability is the probability that the voter will be decisive in creating or breaking an M^{th}–place tie. Note that the voter affects the election result if and only if he or she casts a vote that creates or breaks an M^{th}–place tie. A rational voter will vote for the candidate i that maximizes the $E_i - E_0$ expected-gain equations.

Consider, for example, a three-candidate plurality election. The voter can combine the expected-gain equations for his or her two most preferred candidates (denoted C_1 and C_2) to form the equation $E_1 - E_2$:

$$\begin{array}{rcl} E_1 - E_0 & = & p_{12}(u_1 - u_2) + p_{13}(u_1 - u_3) \\ E_2 - E_0 & = & p_{12}(u_2 - u_1) + p_{23}(u_2 - u_3) \\ \hline \end{array}$$

$$E_1 - E_2 = 2p_{12}(u_1 - u_2) + p_{13}(u_1 - u_3) - p_{23}(u_2 - u_3)$$

By plugging in the voter's utilities (u_1, u_2, u_3) and the pivot probabilities (p_{12}, p_{13}, p_{23}), $E_1 - E_2$ can be calculated and used as follows to determine the candidate for which it is rationally optimal to vote:

$$\begin{array}{l} E_1 - E_2 > 0\text{: vote for } C_1 \\ E_1 - E_2 < 0\text{: vote for } C_2 \end{array}$$

The McKelvey and Ordeshook model is ambiguous about how the voter should vote if $E_1 - E_2 = 0$. For example, this situation occurs when the voter is indifferent between all candidates, that is $u_1 = u_2 = u_3$. When this occurs, the voter cannot expect to be better off voting for C_1 rather than C_2 or vice versa. In this case the voter may defer to a *tie-breaking rule* which might, for example, specify abstention, a random selection, or a bandwagon or underdog strategy. Although unlikely, this situation can also occur when the voter is not indifferent between all candidates. In this case the voter may choose any of the above options or vote for C_1 (the voter's preferred candidate) as a show of support.

The expected-gain equations presented for plurality elections in Equation 4.1 may also be applied to cumulative vote elections [55] and Borda count elections [70] that select a single alternative. In cumulative vote elections the strategic voter should distribute all of his or her votes to the alternative i that maximizes $E_i - E_0$. In Borda count elections the strategic voter should rank the candidates according to their $E_i - E_0$ values. The expected-gain equations may also be used for single non-transferable vote elections (SNTV). In this case they correctly result in rational voters abandoning their most preferred candidates if they are either too far behind or sufficiently far ahead that their victory is assured [34].

Approval voting requires a slightly different approach: strategic voters should always vote for their first choice candidate, never vote for their last choice candidate, and vote for any other candidate C_i if and only if [55]

$$u_i > \frac{\sum_{i \neq j}^{N} u_j p_{ij}}{\sum_{i \neq j}^{N} p_{ij}}$$

The formulation of optimal strategies becomes somewhat more complicated when multiple alternatives are to be selected and voters must vote for $L > 1$ alternatives, as in the limited vote method and plurality rule applied to multiple alternatives. Hoffman suggests that voters consider every possible group of alternatives that might be elected and assign a utility value to each group (this may be accomplished more easily by assigning utilities to each alternative and summing the utilities for each group member to determine the group utility). Each group can then be considered as a distinct alternative and the plurality expected-gain formula may be applied [55]. However, as the number of alternatives increases this approach can result in large numbers of expected-gain equations; for example, 210 equations must be considered when selecting six alternatives from among 10 choices. (This approach is by no means intractable; however, as the number of equations remains of polynomial order.) Another approach would be to apply the plurality expected-gain formula as with SNTV, but vote for the candidates that correspond to the top L expected gains. This approach assumes the alternatives are essentially independent.

In developing our declared-strategy voting system, we assume that rational voters will develop declared strategies consistent with the expected-utility model. Therefore, to accommodate these voters, our system need only accept strategies based on the expected-gain equations for the voting procedure being used (simple plurality, approval, etc.) and the number of candidates under consideration. In fact, we have found that a tie-breaking rule plus a single strategy function in terms of utilities and pivot probability estimates are sufficient for representing the declared strategy of any rational voter. This approach even accommodates those voters who wish to vote despite being indifferent between all candidates, as we shall see in the Section 4.3.

4.2 Pivot Probability Calculations

While the expected-utility model includes a straightforward decision rule, it does not include a method for calculating pivot probabilities. Indeed this calculation appears to be anything but straightforward. Several approaches to this calculation from the literature are presented and analyzed here. These approaches all become more computationally complex

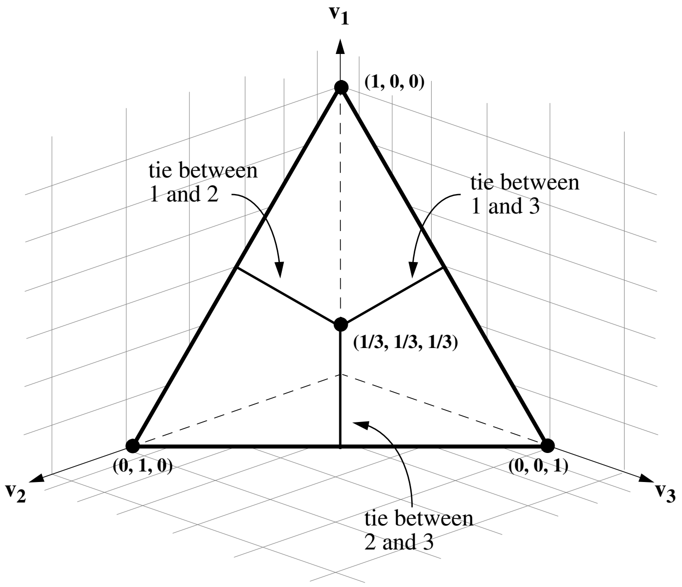

Figure 4.2: A three-dimensional view of the barycentric coordinate system

as the number of election alternatives increases. We introduce a novel approach that does not increase in complexity with the number alternatives.

Before discussing methods for calculating pivot probabilities, it is useful to examine the definition of pivot probability in more detail. As already mentioned, a voter's pivot probability for any two candidates is the probability that he or she will be decisive in making or breaking an M^{th}–place tie between those two candidates. If we assume that for each possible election outcome we can determine the probability associated with the occurrence of that outcome, the pivot probability is the sum of the probabilities associated with each of the possible election outcomes that involve an M^{th}–place tie between those two candidates.

The election outcome space for a three-candidate election can be represented visually as a barycentric coordinate system — an equilateral triangle on the three-dimensional plane $v_1 + v_2 + v_3 = 1$, where v_1, v_2, and v_3 represent the proportions of votes for candidates 1, 2, and 3 respectively. Each of the triangle corners represents one candidate. The closer an outcome point is to a particular corner, the more votes the alternative represented by that corner received. Thus, an outcome point in a corner of the triangle represents a "shut-out" in which one alternative received all the votes, while an outcome point in the geometrical center of the triangle represents a three-way tie. As shown in Figure 4.2, the line segments that bisect the triangle represent two-way winning ties. This model can be extended arbitrarily. For an election with four alternatives the barycentric triangle becomes a solid tetrahedron.

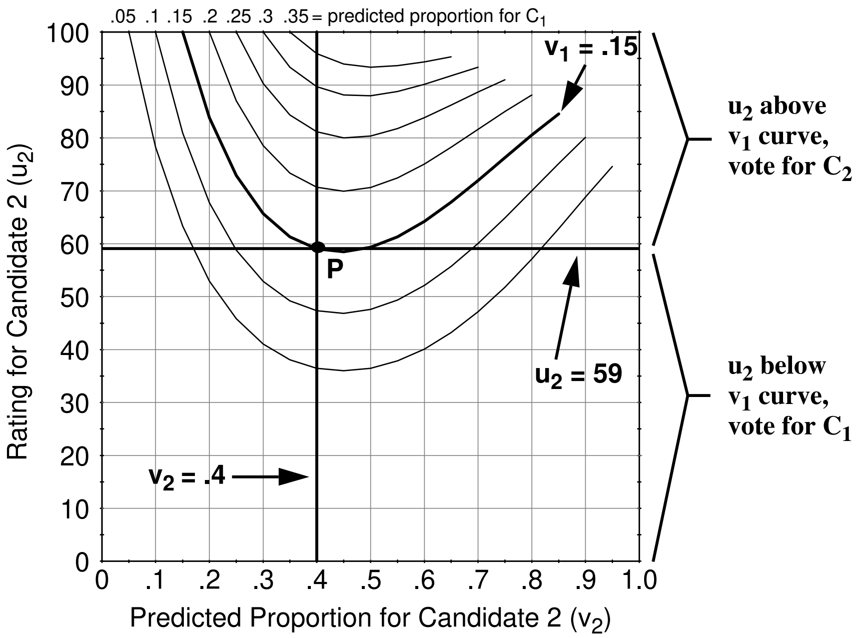

Figure 4.3: Example contour plot

An election with N alternatives can be represented by a simplex polyhedron of $N-1$ dimensions.

Because finding the candidate that maximizes the expected-gain equations (Equation 1) depends on the ratio of the pivot probabilities rather than the actual probabilities themselves, we do not need to find a method for calculating the absolute probability of reaching each point in the outcome space; rather, it is sufficient to find a method of calculating relative probabilities. Thus the methods we examine here do not necessarily result in the probabilities over the entire outcome space summing to 1.

When comparing probability calculation methods and analyzing the suitable of each for use in the expected-utility model of voting, it is helpful to be able to visualize the behavior of the expected-utility model when different probability calculation methods are used. Hoffman [55] includes contour plots in his paper to help illustrate this behavior. To allow visual comparison of our new method with Hoffman's we wrote a computer program to generate contour plots such as the one shown in Figure 4.3. The curves on these plots represent predicted outcome-proportions for a voter's first choice, C_1, in a three-candidate election. Predicted outcome-proportions for a voter's second choice, C_2, are shown along the x-axis. u_2, a voter's utility rating for C_2 is shown along the y-axis. Given a predicted outcome at point $P = (v_1, v_2, v_3)$, a voter should vote for for C_2 if and only if the point (v_2, u_2) lies

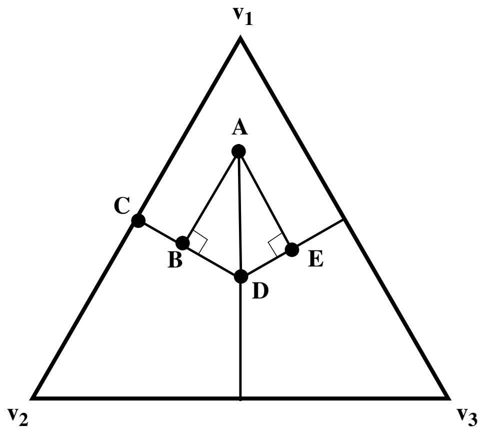

Figure 4.4: Distances used to calculate Black's probability estimates

above the curve for outcome v_1; otherwise the voter should vote for C_1. Consider, for example a predicted outcome of $P = (.15, .4, .45)$. Voters who rate $C_2 \geq 59$ should vote for C_2; voters who rate $C_2 < 59$ should vote for C_1. The plots shown in this chapter all assume normalized ratings such that $100 = u_1 \geq u_2 \geq u_3 = 0$.

We now consider several methods for calculating pivot probabilities.

4.2.1 Simple Distance Methods

One approach to calculating pivot probabilities involves making a prediction about the likely outcome of the election, plotting that outcome as a point on a barycentric coordinate system, and computing the relative distances between that outcome point and the outcome lines for each of the two-way ties. Black [14] used this method in a model of a three-candidate plurality election in which each voter was assumed to be able to make a reasonable prediction about the election outcome based on the results of previous elections, opinion polls, or other data. In batched DSV we can determine the voters' sincere strategies from their utility information, and aggregate these strategies to determine a first-round predicted outcome. Subsequent rounds can use the results of the previous round as a predicted outcome. In ballot-by-ballot DSV, the interim tally of ballots counted so far can serve as the predicted outcome point.

Black assumes that the pivot probability for any pair of candidates is proportional to $1 - d$, where d is the Euclidean distance between the line segment $\overline{CD}$ representing an M^{th}–place tie between those candidates and the predicted outcome point A, as shown in

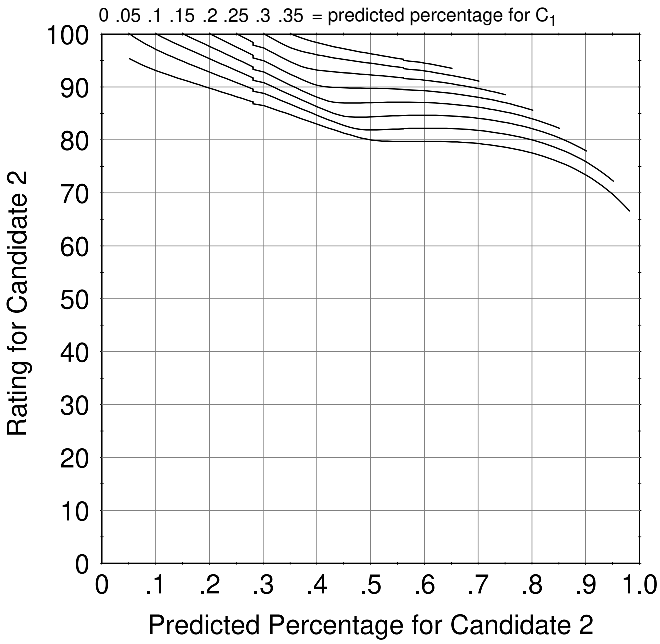

Figure 4.5: Contour plot for 3-candidate plurality election using our simple distance pivot probabilities

Figure 4.4.[1] For p_{12} this distance $|AB|$ is equivalent to the length of the perpendicular line segment $\overline{AB}$ from the predicted outcome point to the first-place tie line $\overline{CD}$.

The distance calculation for $|AB|$ can be expressed as:

$$|AB| = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2}$$

where $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$. The other pivot probabilities can be calculated in a similar manner.

We developed and experimented with a variation on Black's method involves calculating the distance d between the predicted outcome and a given point on a two way tie line. Using this technique, the pivot probability for any two candidates can be found by summing the differences $1 - d$ for every point on the two-way tie line for those candidates. This method makes more intuitive sense than Black's method if one considers what the pivot probabilities really represent.

[1] Note that the example assumes that the predicted outcome ranks candidate 1 in first place, 2 in second place, and 3 in third place. If this predicted ranking does not match the voter's preference ranking, the appropriate substitutions must be made. For example, if the voter had the preference ranking 3 over 2 over 1, the voter's p_{12} would refer to the probability of a tie between candidates 2 and 3.

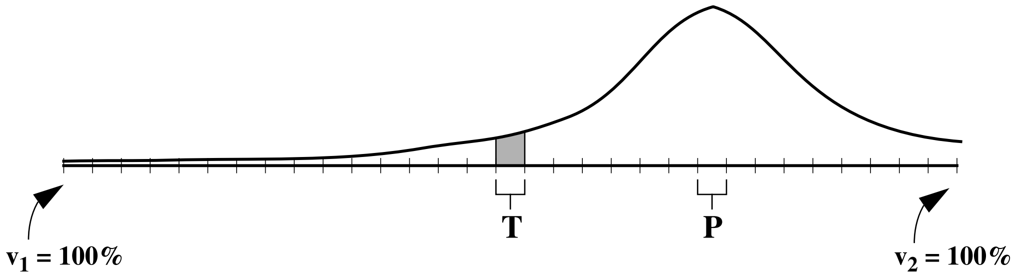

Figure 4.6: Gaussian geometry for a 2-candidate election

One problem with both Black's method and our variation is that they assume that the probability of a tie gets linearly larger the farther away a predicted outcome point is from a two-way tie line. This uniform probability distribution, while simple, does not seem consistent with empirical evidence. For example, as Figure 4.5 illustrates, these methods rarely select the strategy of voting for a second-choice candidate unless the second-choice candidate has a utility rating at least 80% as high as the first-choice candidate. In addition, these methods do not take into consideration the certainty of the predicted outcome point.

4.2.2 Hoffman's Method

Hoffman [55] offers another approach that allows for the modeling of voting schemes other than simple plurality, and assumes a Gaussian distribution of outcomes rather than a uniform distribution. Although Hoffman introduces his approach by describing a three-candidate election, the approach may be applied to elections involving any number of candidates and is most easily understood by considering a two-candidate election first. A two-candidate election with B voters can be modeled as a line segment. This line segment can be divided into B smaller segments, each one vote wide, as shown in Figure 4.6. Each of the smaller line segments represents a possible election outcome. If there are an odd number of voters, there will be a line segment T at the center of the line that represents a tie. If we randomly sample the voters, we can predict an election outcome and represent it somewhere along our line at segment P.

Given P, we are interested in determining the probability that the election outcome will be a tie. As Figure 4.6 illustrates, we can construct a normal distribution curve around the center point of P, given by the formula

$$f(D) = \frac{1}{\sqrt{2\pi}} e^{-D^2/2S^2} \tag{4.2}$$

where D is the distance from each point to (the center of) P, and S^2 is a measure of the uncertainty of the prediction. Hoffman does not specify how S^2 should be calculated, but the "standard error of the estimate" seems to be an appropriate measure of uncertainty. As we shall see, S^2 is an important factor and can affect the outcome of a DSV election dramatically. When using the standard error to calculate S^2 in a ballot-by-ballot DSV system, we multiply it by the finite-population correction factor yielding:

$$S^2 = z_{\alpha/2}^2 \cdot \frac{v_1 \cdot v_2}{b-1}\left(\frac{B-b}{B-1}\right) \tag{4.3}$$

where v_1 and v_2 represent the proportion of votes for each candidate thus far, B represents the total voter population, b represents the number of votes counted thus far, and $z_{\alpha/2}$ represents the area under the standard normal curve for confidence level $1-\alpha$. Thus as the election progresses, b will increase and S^2 will decrease. The fact that uncertainty decreases as the election progresses is consistent with the fact that the amount of information increases.

The above method of calculating uncertainty is useful for ballot-by-ballot DSV, in which the predicted outcome involves a sample that increases in size until it equals the entire population; however, in batch DSV, the predicted outcome point is based on polling the entire population — thus our prediction derives no uncertainty from sampling error. In batch DSV the uncertainty of the prediction is based on not knowing how many voters will find that their optimal strategies are not their sincere strategies.

The simplest way to account for uncertainty in batch DSV would be to select an arbitrary value for S^2. For example, $S^2 = .026$ has the property that as long as a C_1 is predicted to receive at least 5% of the vote, voters will not vote for C_2 unless u_2 is at least 10% as large as u_1 (on a 10 point scale, a voter with $u_1 = 10$ must have $u_2 \geq 1$ in order to vote for C_2). We used this approach in our simulations, described in Chapter 5. We also tried conducting batch DSV simulations with the number of rounds equal to the number of voters and adjusting S^2 according to Equation 4.3, letting b equal the number of rounds run so far. However this approach generally resulted in the same election outcome as was produced using a constant S^2 equal to the value produced by Equation 4.3 when $b = B$.

Another approach to calculating S^2 would be to develop a formula based on some aggregation of the utilities submitted by the voters — perhaps taking into account the percentage of voters for whom it might never be optimal to voter for C_2. Such a calculation is also likely to be somewhat arbitrary, however. Still another approach would be to assign random values to S^2 within a reasonable range (say $.001 - .5$). Finally, voters could select their own uncertainty values based on their attitudes toward risk, formulated perhaps

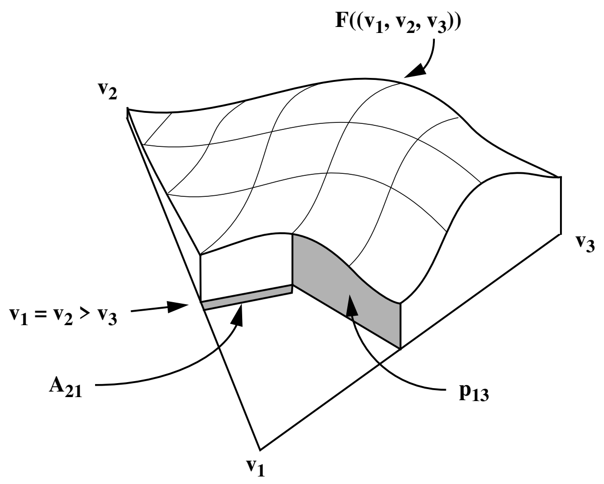

Figure 4.7: Hoffman's geometry for a 3-candidate election

as a function of the "maturity" of the election: number of rounds, position in the voting sequence, etc.

Regardless of what method is used to calculate S^2, the probability that the election outcome will involve a first-place tie between candidates 1 and 2 can be computed as follows:

$$p_{ij} = \frac{1}{\sqrt{2\pi}} \int_{T-U/2}^{T+U/2} f(D)dx$$

where $U = 1/B$ (the length of a one-vote wide line segment). That is, the probability of a tie is proportional to the area under the segment of the normal curve that lies above the tie segment T.

Hoffman illustrates his approach for a three-candidate election. As shown in Figure 4.7, Hoffman specifies the region A_{ij} as the portion of the outcome triangle in which candidate i loses to candidate j by one vote (or less in systems that allow fractional votes). He defines the pivot probability as "the probability that the election result (v_1, v_2, v_3) lies in the region A_{ij}." Thus p_{ij} can be expressed as:

$$p_{ij} = \iint_{A_{ij}} f(D)$$

where D is the distance from the predicted outcome point to a point in the region A_{ij}. Because A_{ij} is only one vote wide, p_{ij} can be approximated by using Simpson's rule or another numerical integration technique to integrate over the outcome points for which $v_i = v_j > v_k$. Irrespective of the number of candidates N, first-place ties can occur only when

$1/N \leq v_i = v_j \leq 1/2$. Values of v_i and v_j greater than $1/2$ would be impossible because $v_i + v_j > 1$; and values of v_i and v_j less than $1/N$ would indicate that at least one other candidate has a higher proportion of the vote. Values of v_i and v_j between $1/N$ and $1/3$ may or may not indicate first-place ties, depending on the outcome proportions for the other candidates. However, when $1/3 \leq v_i = v_j \leq 1/2$, v_i and v_j must be in a first-place tie. Thus, for a three-candidate election, p_{ij} can be approximated by:

$$p_{ij} \approx \int_{1/3}^{1/2} f(D)dx$$

where x represents the proportion of votes for candidates i and j at a given outcome point. Note that when $N > 2$, the standard error of the estimate becomes quite difficult to calculate. We use the approximation[2]

$$\frac{v_i(1-v_i) + v_j(1-v_j) + \cdots + v_N(1-v_N)}{(N-1)(b-1)} \tag{4.4}$$

to calculate the standard error of the estimate for $N > 2$.

A four-candidate election is modeled as a solid tetrahedron. However, finding the outcome region in which first-place ties occur is not as easy in four dimensions as it is in two and three. As discussed above, first-place ties may occur in regions where $1/N \leq v_i = v_j \geq 1/3$, depending on the proportion of votes received by each of the other candidates. With four candidates, the outcome region in which candidates i and j tie for first place is a kite-shaped section of the plane. The probability of an outcome occurring in this region can be approximated by:

$$p_{ij} \approx \int_{1-3x}^{x} \int_{1/4}^{1/3} f(D)dx\,dy + \int_{0}^{1-2x} \int_{1/3}^{1/2} f(D)dx\,dy$$

where x represents the proportion of votes for candidates i and j, and y represents the proportion of votes for candidate k at a given outcome point.

Hoffman's approach is appealing because it does not assume a uniform probability distribution and because it takes into account the uncertainty associated with the predicted outcome. Indeed, this uncertainty proves to be a significant factor in calculating a voter's optimal strategy. Figure 4.8 shows the contours we have calculated for S^2 values of .02 and $.05$. The figure illustrates that the more certain the prediction, the more willing voters should be to vote for C_2 rather than C_1. For example, given a predicted outcome P of $(.15, .4, .45)$

[2]Thanks to Professor Edward Spitznagel for suggesting this approximation.

the optimal strategy when $S^2 = .05$ is to vote for C^2 if $u_2 \geq 59$. However under the same circumstances but with $S^2 = .02$, it is optimal to vote for C_2 if $u_2 \geq 24$.

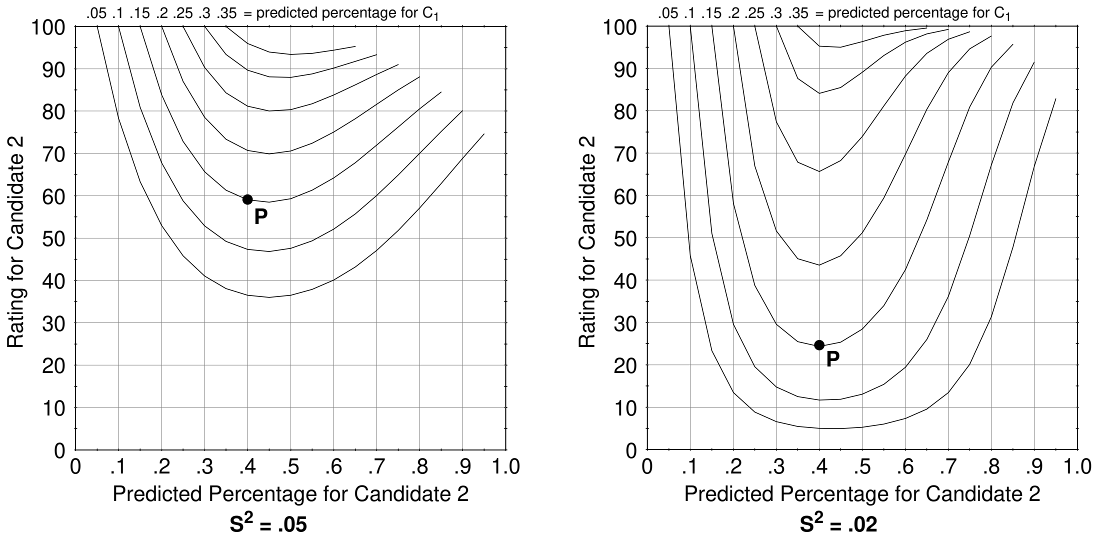

Figure 4.8: Contour plot for 3-candidate plurality election, Hoffman's method pivot probabilities

4.2.3 2-D Gaussian Method

A drawback to Hoffman's approach is that the pivot probability calculations increase in complexity with the number of candidates. For elections involving many candidates, these calculations can be quite difficult and time consuming. The reason for the increasing complexity is that Hoffman's method requires the projected outcome for all candidates to be considered in every pivot probability calculation. However, we have discovered that the only projected outcomes crucial to the pivot probability calculation are the outcomes for the candidate predicted to win and for the two candidates whose pivot probability is being computed. Furthermore, a simplifying assumption allows us to calculate pivot probabilities for N candidate elections using the predicted outcomes for just two candidates.

To calculate p_{ij} using our *2-D Gaussian method,* we being by plotting the predicted outcomes for i and j along an outcome line and constructing normal distribution curves around each outcome point, as shown in Figure 4.9. In ballot-by-ballot DSV it is important that the S^2 values used to construct the normal curves decrease as the election progresses. We can simplify our S^2 calculation while still ensuring that S^2 decreases by substituting

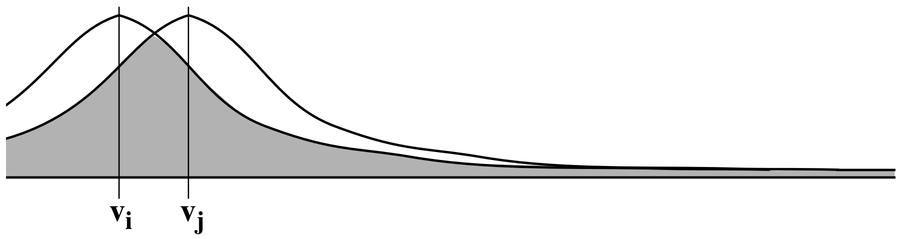

Figure 4.9: Predicted outcomes for candidates i and j

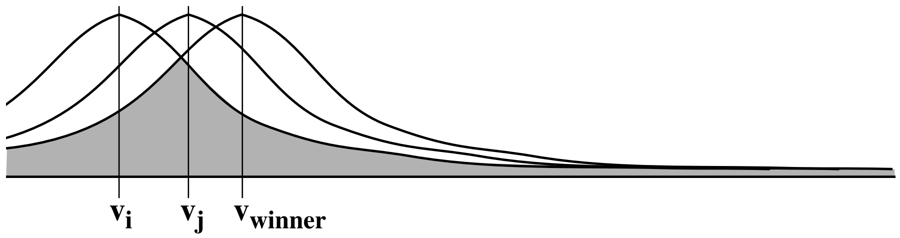

Figure 4.10: Geometry for calculating pivot probability using 2-D Gaussian method

an average value for the numerator in the standard error approximation presented in Equation 4.4:

$$S^2 = \frac{(\frac{1}{N})(1-\frac{1}{N})}{b-1}\left(\frac{B-b}{B-1}\right)$$

Thus, as a ballot-by-ballot DSV election progresses, the normal curves will become increasingly narrower. However all the curves calculated for a given ballot will have the same width.

The shaded area in Figure 4.9, where the curves intersect, represents the probability that the final outcome will involve a tie between i and j. But not all ties are winning ties. If our election involved only three candidates we could declare the pivot probability to be the section of the intersection area representing outcomes where i and j each receive between $1/3$ and $1/2$ of the votes. However, elections involving four or more candidates cannot be represented this simply. To solve this problem we plot the outcome point for the predicted winner on our outcome line and construct a normal curve around it. As illustrated in Figure 4.10, the pivot probability is the intersection of all three curves.

Note, that when either i or j is the predicted winner, only two curves need be plotted. Also note, that because the curves all use the same value of S^2, the intersection of three curves is equal to the intersection of the left-most and right-most curve. Thus, to compute

p_{ij} we need only determine the area of the intersection of the curve for the predicted winner and either i or j, whichever is predicted to receive fewer votes.

To compute the area of the intersection we must first determine the point at which the two normal curves intersect. The point of intersection I can be calculated:

$$I = \frac{v_{winner}^2 - v_{min(i,j)}^2}{2(v_{winner} - v_{min(i,j)})}$$

where v_{winner} is the predicted outcome for the predicted winner and $v_{min(i,j)}$ is the predicted outcome for either i or j, whichever is predicted to receive fewer votes. The pivot probability p_{ij} is then calculated:

$$p_{ij} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{I} e^{-D^2/2S^2} dx + \frac{1}{\sqrt{2\pi}} \int_{I}^{\infty} e^{-D^2/2S^2} dx$$

where $D = x - v_{winner}$. Because there is no region in which ties are guaranteed to occur, we assume the curve is infinite in both directions. However, only half the area need be computed, as the curve is symmetrical about the intersection point.

The above integral can be approximated using Simpson's rule or another numerical integration technique. However, as S^2 decreases, Simpson's rule requires increasingly more steps to converge. A more efficient approach is to use a table of areas under the standard normal curve or a computer function that returns the values in such a table. In this case a change of variable must be performed to remove S^2 from the exponent. The pivot probability is then calculated:

$$p_{ij} = \frac{2S}{\sqrt{2\pi}} \int_{-\infty}^{I/S} e^{-D^2/2}\, dx$$

In our simulations we use the `erf` function from the C math library. However, the `erf` function is not intended for use with the large upper bound values that occur when S^2 gets very small. When our computations using the `erf` function return 0, we approximate the area as

$$p_{ij} \approx \left(\frac{2S}{D}\right) \frac{1}{\sqrt{2\pi}} e^{D^2/2S^2}$$

where $D = |I - v_{winner}|$.

Our 2-D Gaussian method results in contour plots (shown in Figure 4.11 quite similar to those produced using Hoffman's method. The most obvious difference between the results of these two methods can be seen in the effects of varying S^2. An S^2 value approximately 0.4 times smaller than the one used in Hoffman's method must be used in the 2-D Gaussian method to produce similar results.

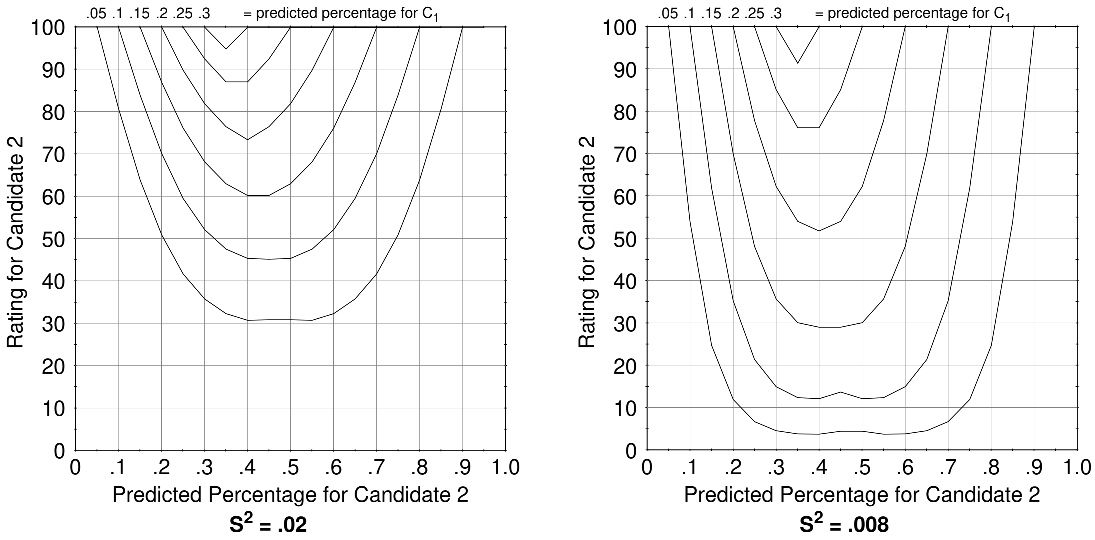

Figure 4.11: Contour plot for 3-candidate plurality election, 2-D Gaussian method pivot probabilities

Another difference can be observed by comparing the upper right-hand corners of Figures 4.8 and 4.11. When a voter's second-choice is predicted to win by a large margin the 2-D Gaussian method produces pivot probabilities that will result in a vote for the voter's first choice, regardless of u_2. Under the same circumstances Hoffman's method produces pivot probabilities that may result in a vote for the voter's second choice, depending on u_2. In this situation, the voter's first choice has practically no chance of winning and the voter's second choice has practically no chance of loosing. Therefore, the voter cannot influence the outcome with his or her vote, and thus, we believe, should vote sincerely. Hoffman's method and the 2-D Gaussian method indicate different strategies in this situation due to the fact that for a three-candidate race Hoffman's method indicates a higher probability of a winning tie between third-place and first-place candidates than between third-place and second-place candidates, while our method indicates that the probability of a winning tie is the same for these two pairs of candidates. Because the number of voters who must change their votes to achieve a winning tie in each of these two situations is identical, we believe our method is a more accurate model of pivot probabilities involving third-place candidates.

4.3 Tie-Breaking Rules

Because the expected-utility model fails to specify a strategy when voters are indifferent between alternatives, it can be extended easily to model voters who wish to base their strategies on factors other than their preferences for the alternatives. These voters may submit equal utility values for all candidates and have their votes based solely on their tie-breaking rules. This is consistent with our notion of rationality assuming that these other factors so outweigh any difference in a voter's preference between alternatives that the voter is essentially indifferent between them.

One situation where tie-breaking rules might prove useful involves voters who are indifferent between the candidates running in their party's primary but want to vote for whichever candidate is most likely to win in order boost the winner's share of the vote and make their party appear more unified. These voters might use a bandwagon strategy.

In theory, tie-breaking rules could be arbitrarily complex and involve many variables. For simplicity we limit tie-breaking rules to first-order functions of the predicted outcome and the maturity of the election.

4.4 The Decision-Theoretic Strategy Formulator

In this chapter we have seen how the expected-utility model can be used to produce optimal strategies based on a voter's preferences and predictions about the outcome of an election. A voter's preferences may be represented as utility values on a finite scale, say from 0 to 10. In addition, voters who are indifferent between alternatives must also supply tie-breaking rules. Depending on the pivot-probability method used, we might also ask voters to supply a measure of their attitude towards risk. Given this information, a computer can calculate the optimal decision-theoretic strategy for each voter and submit it to the strategy aggregator, as shown in Figure 4.12. Thus, voters need not understand the process of formulating an expected-utility maximization function to use this system.

A strategy formulator that uses our 2-D Gaussian method for calculating pivot probabilities formulates an optimal strategy for an N–alternative plurality-rule election as shown in Figure 4.13.

Note that we ask voters to submit their sincere utility values for all candidates. Our system calculates the optimal decision-theoretic strategy based on these values, enabling voters to vote strategically while simultaneously revealing their sincere preferences. Thus it is generally advantageous for voters to submit their sincere utility values. However, as we

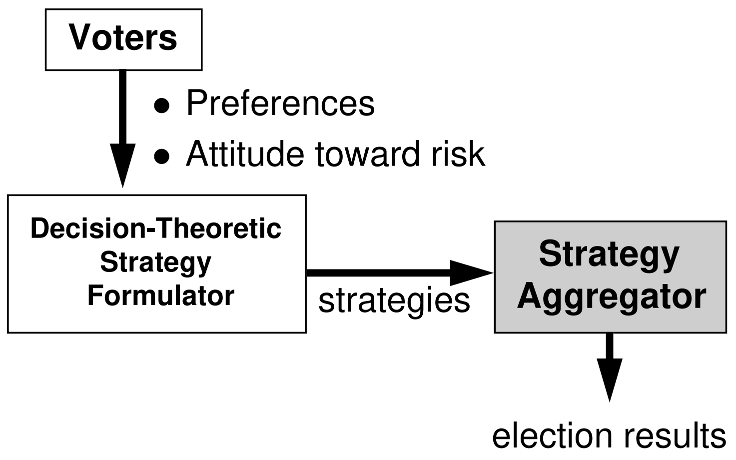

Figure 4.12: A model of declared-strategy voting with decision-theoretic strategy formulator

discuss later, occasionally voters who have the necessary knowledge to formulate sophisticated strategies might not find it to their advantage to reveal their sincere utility values (and will act accordingly).

N = number of candidates
B = number of voters (= number of ballots)
b = number of ballots counted thus far
u_i = voter's utility for alternative i (from voter's ballot)
v_i = predicted outcome proportion for alternative i

Optimal strategy: select the alternative i that maximizes the expected gain equation

$$E_i - E_0 = \sum_{i \neq j}^{N} (u_i - u_j) \frac{2S}{\sqrt{2\pi}} \int_{-\infty}^{I/S} e^{\frac{-D^2}{2}} \, dx$$

where

$$D = x - v_{winner}$$

$$I = \frac{v_{winner}^2 - v_{min(i,j)}^2}{2(v_{winner} - v_{min(i,j)})}$$

In batch DSV S is a constant. In ballot-by-ballot DSV

$$S = \sqrt{\frac{(\frac{1}{N})(1 - \frac{1}{N})}{b - 1} \left(\frac{B - b}{B - 1} \right)}$$

Figure 4.13: Summary of formulas for optimal plurality-rule strategy with 2-D Gaussian pivot probabilities

Chapter 5

Evaluation of Declared Strategy Voting

The theory of voting is not worth bothering with unless it is going to be applied.
Michael Dummett, 1984 [40]

While DSV is quite interesting from a theoretical standpoint, the theory would be of little consequence if it could not be applied to real voting situations. In this chapter we evaluate the behavior of declared-strategy voting systems and assess their suitability for real-world group decision-making. In Section 5.1 we present an acceptability study designed to assess attitudes about DSV. In Section 5.2 we present the results of our DSV simulations. In Section 5.3 we present a case study of the 1992 Presidential election and discuss the impact DSV might have had on that election. In Section 5.4 we return to the evaluation criteria presented in Chapter 2, applying them to DSV.

5.1 Acceptability Study

In this section we present a study we conducted to determine the acceptability of DSV by members of a relatively well-educated population. Clearly a more general study is necessary to determine the acceptability of DSV for public elections. However, this limited study provides insights into some of the reactions to DSV that are likely to be encountered in any voter population.

5.1.1 Survey Design

One hundred seventy-one Washington University students were surveyed to gauge their attitudes towards declared-strategy voting and to collect preference data for use in simulations. Survey forms were distributed over a two-day period during a Summer semester in first-semester physics and chemistry classes in the College of Arts and Sciences and in MBA classes in the John M. Olin School of Business. In each class, the survey administrator collected the surveys as students completed them, about 5 to 10 minutes after they were distributed. The survey instrument is reproduced in Appendix A.

A total of 96 business students and 75 science students completed the survey. The business students were mostly graduate students, often with professional experience. The science students were mostly undergraduates majoring in science or engineering, or planning to attend medical school.

5.1.2 Survey Results

The first question asked whether respondents had attended school full time during the Spring 1996 semester. All respondents who had attended school full time during the Spring 1996 semester were classified as full-time students. The remainder were classified as part-time students. The distribution of full-time and part-time students was as follows:

Question 1

	Business	Science	All
Percent Full Time	43	89	63
Percent Part Time	57	11	37

Overall there were few significant differences between the responses of full-time and part-time students or between the responses of business and science students.

Questions 2 and 3 collected preference data for use in DSV simulations. These simulations will be discussed in the next section.

Question 4 asked respondents whether they had voted in a student government election during the 1995-96 school year. Only 25% responded affirmatively. As the following table illustrates, most of the student government voters were science students. This may be attributable to the fact that at Washington University, undergraduates are more likely to vote in student government elections than graduate students, and more of the science students were undergraduates.

Question 4

	Business	Science	All
Percent Who Voted in Student Government Election	7	48	25
Percent Who Did Not Vote in Student Government Election	93	52	75

Question 5 asked respondents whether they were registered to vote in the United States. As the following table illustrates, business students were more likely to be registered than science students, possibly due the fact that the business students were generally older than the science students.

Question 5

	Business	Science	All
Percent Registered	82	59	72
Percent Eligible But Not Registered	2	17	9
Percent Ineligible	16	24	19

Question 6 asked respondents to rate five potential US Presidential candidates on a scale of 0 to 10. These rankings were subsequently used in the simulations described in the next section. In addition this question provided a context for the remaining questions.

Questions 7 and 8 were introduced as follows:

> Most elections in the US use a voting system called *plurality rule*. In this system voters mark a single choice on their ballots, and the candidate with the most votes wins. Imagine that instead of using plurality rule, the 1996 US Presidential ballot asks voters to rate each candidate as you did in Question 6. The candidate with the highest rating would be a voter's first choice. One way to tally this kind of ballot would be to have a specially programmed computer analyze each ballot and cast a vote that will maximize the influence of that ballot.

Question 7 then described the following scenario:

> Suppose that on the ballot described above your 1st choice ends up coming in last, as shown in the table to the right. However, your 2nd choice comes in first. Your 5th choice comes in a close second. Now suppose that when your ballot was counted, the computer cast your vote for your 2nd choice because it was unlikely that your 1st choice could win and your 2nd choice needed your vote to beat your 5th choice. How satisfied would you be with the performance of the computer in this situation?

On a scale of 1 to 5, where 1 indicated extreme dissatisfaction and 5 indicated extreme satisfaction, the mean response was 2.85 with a standard deviation of 1.26.

Question 7

	1	2	3	4	5	Mean	Median	SD
Percentage of business students	15	21	29	25	10	2.93	3	1.21
Percentage of science students	24	23	17	27	9	2.73	3	1.33
Percentage of all respondents	19	22	24	26	9	2.85	3	1.26

Question 8 described a scenario identical to the one described in question 7, with the exception that the 5th choice ends up winning. On a scale of 1 to 5, where 1 indicated extreme dissatisfaction and 5 indicated extreme satisfaction, the mean response was 2.50.

Question 8

	1	2	3	4	5	Mean	Median	SD
Percentage of business students	29	13	32	19	8	2.64	3	1.29
Percentage of science students	33	20	30	13	3	2.32	2	1.15
Percentage of all respondents	30	16	31	16	6	2.50	3	1.23

The results of questions 7 and 8 indicate that overall respondents were slightly dissatisfied with the way the computer cast their votes. The fact that there was more dissatisfaction expressed in response to question 8 (where the 5th choice won) than in response to question 7 (where the 2nd choice won) suggests that respondents had difficulty evaluating the voting procedure separately from the election outcome. Further analysis reveals 55% of respondents indicated the same level of satisfaction in response to both questions, 32% indicated more satisfaction in response to question 7, and 12% indicated more satisfaction in response to question 8. It is not clear why a respondent would be more satisfied with the performance of the computer in question 8 than in question 7. It is likely that those who gave that type of response did not fully understand the question. However, removing these respondents from the analysis of this or other questions does not significantly impact the results.

Question 9 asked “How certain would you want to be that your first choice was not going to win before you supported the computer voting for your second choice?” Overall, 74% of respondents supported the computer voting for their second choice under some conditions. The results of this question showed some significant differences between these groups of business and science students, with the science students more reluctant to let a computer decide to vote for their second choice.

Question 9

	Business	Science	All
Percent who never want computer to vote for 2nd choice	24	30	26
Percent who want to be 100% certain	10	26	17
Percent who want to be at least 90% certain	19	17	18
Percent who want to be at least 80% certain	23	10	17
Percent who want to be at least 70% certain	12	9	10
Percent who want to be at least 60% certain	3	4	4
Percent who want to be at least 50% certain	3	3	3
Percent who want the computer to vote for 2nd choice whenever 2nd choice has better chance of winning than 1st choice	6	1	4

Note that the last response option for question 9 is essentially the bandwagon option discussed in Chapter 4.

Question 10 asked "How easy or difficult do you find it to understand declared-strategy voting?" On a scale of 1 to 5, where 1 indicated extremely difficult and 5 indicated extremely easy, the mean response was 3.52.

Question 10

	1	2	3	4	5	Mean	Median	SD
Percentage of business students	4	14	24	37	20	3.55	4	1.09
Percentage of science students	3	15	30	33	18	3.49	4	1.04
Percentage of all respondents	4	14	27	35	19	3.52	4	1.07

Question 11 asked voters to compare the fairness of DSV and plurality voting. This question was designed to get a gut-feeling reaction, and no definition of fairness was given. On a scale of 1 to 5, where 1 indicated that DSV is much more fair and 5 indicated that DSV is much less fair, the mean response was 3.25. Business students were more likely than science students to find DSV more fair than plurality.

Question 11

	1	2	3	4	5	Mean	Median	SD
Percentage of business students	5	30	32	23	10	3.02	3	1.07
Percentage of science students	0	20	27	32	21	3.55	4	1.04
Percentage of all respondents	3	25	30	27	15	3.25	3	1.09

Questions 12, 13, and 14 asked respondents how they would feel if DSV were adopted for US Presidential elections, student government elections, and the tallying of university surveys, respectively. Respondents had the strongest feelings about Presidential elections, with over 40% expressing indifference (a 3 response) about the use of DSV for student government elections or surveys. On a scale of 1 to 5, where 1 indicated extreme dissatisfaction and 5 indicated extreme satisfaction, the means for the three questions were 2.52, 2.93, and 2.88.

Questions 12, 13, 14

	1	2	3	4	5	Mean	Median	SD
Percentage of all respondents for Question 12 (US Presidential elections)	25	25	25	19	5	2.52	2	1.20
Percentage of all respondents for Question 13 (student government elections)	16	13	43	19	9	2.93	3	1.14
Percentage of all respondents for Question 14 (university surveys)	19	12	41	20	8	2.88	3	1.17

Overall 33% of respondents indicated less satisfaction about the prospect of using DSV for Presidential elections than for student government elections and 34% indicated less satisfaction about the prospect of using DSV for Presidential elections than for surveys. It is not surprising that respondents had stronger opinions about the Presidential election than about student government elections, especially considering that the majority of students did not report voting in a student government election in the past year. It is somewhat surprising that students were slightly less satisfied by the prospect of using DSV for surveys than for student government elections. However, this is likely due to the fact that there was more indifference about student government elections than about surveys. Overall, students were slightly dissatisfied about the prospect of using DSV in any of these contexts. This indicates that voter education would be essential before this population was likely to accept DSV (indeed extensive voter education has been necessary to adopt most new types of voting systems). However, it also indicates that this population is not completely set against using DSV.

At the end of the survey, respondents were asked "In your opinion, what are the advantages of declared-strategy voting?" and "In your opinion, what are the disadvantages of declared-strategy voting?" Only 39% of the respondents answered one or both of these questions, and it was difficult to tell what some of the respondents meant by what they wrote.

However, we were able to group most of the responses into categories according to common themes, many of which have also come up in informal conversations about declared-strategy voting. The following themes were each mentioned at least three times by the survey respondents. Some respondents mentioned more than one theme in their responses. Note that some respondents specifically listed “no advantages” or a similar phrase in their response to question 15; thus, we include this as a response theme.

Question 15

Advantages	**Mentions**
Individual votes are likely to be more effective	15
No advantages	10
Takes into account complete preference ranking and strength of preferences	7
Elects “compromise” candidate or more generally acceptable candidate than plurality	4
More efficient than run-off elections	3
Prevents the vote from being “split” by third-party candidates	3
Results are more meaningful	3

Question 16

Disadvantages	**Mentions**
Use of randomness and statistical trend may not always result in same outcome	10
May force voters to vote for a second-choice alternative when they really want to vote only for their first choice (or they may not have a second choice)	10
Uses computers	8
Too complicated, too difficult to understand	7
Undemocratic	5
May be manipulated	5
Unfair	4

With the exception of those respondents who were convinced that there are no advantages to DSV, most of the respondents who listed advantages were correct in their observations. While a fairly low number of respondents recognized the advantages of DSV, it is encouraging that some respondents were able to recognize these advantages given only limited information about DSV.

One of the most frequently mentioned disadvantages of DSV is indeed one of the most significant problems with ballot-by-ballot DSV (the only type of DSV described in the survey): a given set of ballots does not always produce the same outcome. This problem is discussed in more detail later in this chapter. Although this problem may not be critical from a theoretical standpoint (as we shall explain later) it is important to realize that it may be viewed by voters as a major problem. This problem may serve to limit the applications in which ballot-by-ballot DSV is acceptable unless it can be resolved or explained to the satisfaction of the voters.

Many of the other disadvantages mentioned by survey respondents can be attributed to the respondents' limited understanding of DSV. For example, respondents complained that DSV might force voters to vote for a second choice even if they did not have a second choice or only wanted to support a first choice. However, DSV does not require voters to support more than one choice (or even support one choice). Voters who wish to vote only for their sincerely preferred candidate may give zero ratings to the other candidates. This was not made clear on the survey form.

Some respondents seem to equate democracy with simple majority rule voting. This misunderstanding came up many times during informal discussions with acquaintances as well. Our observations indicate that many people believe that democracy requires simple majority rule voting, and that plurality rule is the same thing as majority rule. They believe that anything other than majority or plurality rule voting would be undemocratic and unconstitutional. They are often surprised to learn that plurality rule voting may produce an outcome that does not have the support of the majority, and that most democratic countries do not use plurality rule voting to elect their representatives. They are also surprised to learn that other voting systems may be enacted without changes to the United States Constitution or the constitutions of the individual states. Clearly before DSV or any alternative voting system is adopted, the prospective electorate would have to be generally educated about voting systems.

Some respondents complained that DSV is susceptible to manipulation. One respondent, apparently somewhat familiar with voting theory, cited Arrow's theorem. However, most respondents did not seem to realize that the voting systems currently used are much more susceptible to manipulation than DSV.

Some respondents viewed the fact that DSV is tallied by a computer as a disadvantage. These respondents stated that they were concerned about the possibility of computer errors or that they did not trust a computer to make a decision on their behalf. These respondents may not be aware that computers are used to tally ballots in the majority of US elections. While the complicated nature of DSV increases the likelihood that an error might be introduced into the vote tallying program, DSV should not suffer from significantly more computer errors than traditional voting methods. Public education may help alleviate such fears.

5.2 Simulation

5.2.1 Simulator Design and Experiment Methodology

We wrote a declared-strategy voting simulator program to study the impact of DSV on election results. The simulator was written in C and runs on a Unix platform. Key parts of the DSV simulator source code are reproduced in Appendix B.

Our DSV simulator can simulate both ballot-by-ballot and batch mode DSV. It offers a variety of methods for calculating pivot-probabilities and uncertainty.

We implemented only one tie-breaking rule in our simulator: a bandwagon rule with persistent ties resolved randomly. Thus, all indifferent voters were assigned a strategy in which they vote for whichever candidate is winning or, in the event of a first-place tie, they select randomly from among those tied for first-place.

For each simulation, we calculated a sincere poll result in addition to a DSV result. The sincere poll result was calculated by casting a vote for the alternative rated highest by each voter. A random selection was made for voters who did not indicate a unique first choice.

We used our 2-D Gaussian method for calculating pivot probabilities in all of the DSV simulations reported here. Other simulations using Hoffman's method resulted in similar outcomes, but with less strategic voting (probably due to differing affects of S^2 in the two methods).

Batch DSV simulations were run with two arbitrary values of S^2: $.001$ (which we shall hereafter refer to as "small S^2") and $.05$ (which we shall hereafter refer to as "large S^2"). Batch simulations were run for as many rounds as necessary for an equilibrium to be

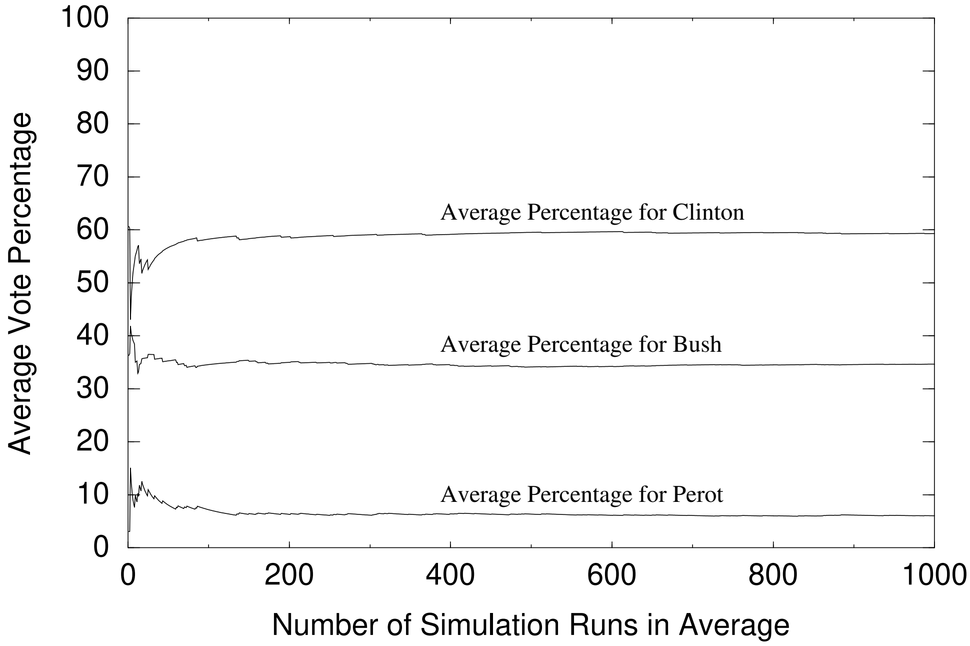

Figure 5.1: Running average for ballot-by-ballot DSV simulation of 1992 US Presidential election

reached. An equilibrium was reached in all simulations without the need for an arbitrary cut-off.

Ballot-by-ballot simulations were run 1000 times on each data set. The ballots in each set were shuffled randomly before each run. The results reported are the average results over all the runs. The running average for most of our simulations became fairly stable (varying by less than 1%) after the results of a few hundred simulations were included in the average. For example, Figure 5.1 shows the running average for ballot-by-ballot DSV simulations using data from the 1992 National Election Study (to be discussed in Section 5.3).

5.2.2 Simulation Results

University Lecture Series Survey Simulation

We simulated the selection of topics for a university lecture series using data collected from 162 respondents who answered question 2 of our survey. The respondents were asked to rate their interest in each of ten possible lecture topics (anthropology, gender issues, health and medicine, history, journalism/media, literature, politics, race relations, science, and sports) on a scale of 0 to 10. As shown in the table below, the sincere poll indicated that science was the most popular topic with 20.4% of the vote, followed closely by sports with 19.1%. Health and politics each received over 10% of the vote as well. Batch DSV also resulted in a victory for science, followed by sports, health, and politics. A decision-maker who could select several lecture topics for inclusion in the lecture schedule would be wise to select these four.

University Lecture Series Survey Simulation

	A	GI	H/M	H	J/M	L	P	RR	Sc	Sp
Sincere poll	7.4	5.6	16.0	3.7	5.6	3.1	12.3	6.8	20.4	19.1
Batch DSV (large S^2)	1.2	3.1	17.9	1.9	1.2	3.7	9.3	0.6	42.0	19.1
Batch DSV (small S^2)	0.0	0.6	6.2	0.0	0.0	0.0	1.2	0.0	63.6	28.4
Ballot-by-ballot DSV	0.2	2.4	30.0	0.8	0.5	0.7	9.4	0.6	33.9	21.4
Percent wins	0.0	0.0	45.5	0.1	0.0	0.0	0.5	0.0	47.0	6.9

The ballot-by-ballot DSV results indicated that science and health were significantly more popular than sports or politics when preferences beyond each voter's first choice were considered. Science averaged 33.9%, winning the ballot-by-ballot DSV election 47% of the time. Health averaged 30.0%, winning 46% of the time. Sports averaged 21.4%, winning 7% of the time, and politics averaged 9.4%, winning less than 1% of the time. An analysis of the individual ballot-by-ballot simulations indicated that in the simulations where science won, health received a low percentage of votes, and in the simulations where health won, science received a low percentage of votes. This indicates that the respondents who were interested in science were likely to be almost as interested in health, and visa versa (thus both groups could be satisfied by the selection of either lecture topic). This observation could be made because DSV allowed those who preferred health and science to pile on to whichever of these topics was more likely to win rather than splitting their votes. Thus DSV is helpful to both the survey respondents and the decision-maker in this situation.

The batch DSV results are less useful to a decision-maker. When S^2 is very small, batch DSV tells us only that science is the most popular topic. When S^2 is large, batch DSV shows us that science is a more popular second choice than the other leading contenders.

Course Times Survey Simulation

We simulated the selection of a course time using data from the 159 respondents who answered question 3 of our survey. The respondents were asked to rate their interest in taking a course at each of seven possible times of day (8:00 am, 10:00 am, 12:00 pm, 2:00 pm, 4:00 pm, and 8:00 pm) on a scale of 0 to 10. As shown in the table below, the sincere poll indicated that 8:00 am was the most popular time, followed closely by 6:00 pm. However ballot-by-ballot DSV showed that 6:00 pm would actually please more people. Under ballot-by-ballot DSV, 6:00 pm won with an average of 42.3%, while 8:00 am came in second with an average of 31.6%. Indeed an analysis of respondents' ratings for just these two time slots indicates that 66% preferred 6:00 pm to 8:00 am. It is also useful to note that while 8:00 am actually won more ballot-by-ballot DSV elections than 6:00 pm, 6:00 pm came in a close second in most of the elections in which 8:00 am won, while 8:00 am was nearly shut out in most of the elections in which 6 pm won. This is due to the fact that most of the respondents who preferred 6 pm found most other times completely unacceptable, while the respondents who preferred 8 am found other times acceptable as well.

Course Times Survey Simulation

	8 am	10 am	12 pm	2 pm	4 pm	6 pm	8 pm
Sincere poll	34.3	8.8	6.9	2.5	8.2	28.9	8.2
Batch DSV (large S^2)	38.4	3.1	8.8	0.1	6.3	40.3	2.6
Batch DSV (small S^2)	47.8	0.0	4.4	0.0	5.0	42.8	0.0
Ballot-by-ballot DSV	31.6	8.2	10.1	0.9	5.0	42.3	1.9
Percent wins	62.2	2.2	3.6	0.0	0.0	28.6	0.0

Hypothetical US Presidential Election Simulation

We simulated a hypothetical US Presidential election using data from the 164 respondents who answered question 6 of our survey. The respondents were asked to rate their approval for each of five hypothetical Presidential candidates (Bill Clinton, Bob Dole, Ross Perot, Colin Powell, and Dan Quayle) on a scale of 0 to 10. As shown in the table below, the sincere poll elected Clinton, with Powell in second place. However simulations using ballot-by-ballot DSV and batch DSV with small S^2 values elected Powell with Clinton in second place. This is due to the fact that many Dole, Perot, and Quayle supporters also supported Powell. The ballot-by-ballot simulations were fairly stable for this election, electing Powell 90.2% of the time.

Hypothetical US Presidential Election Simulation

	Clinton	Dole	Perot	Powell	Quayle
Sincere poll	43.2	14.8	4.3	32.7	4.9
Batch DSV (large S^2)	43.8	11.7	2.5	39.5	2.5
Batch DSV (small S^2)	31.5	3.1	0.0	65.4	0.0
Ballot-by-ballot DSV	31.3	8.9	0.8	57.3	0.5
Percent wins	8.1	1.6	0.0	90.2	0.0

Further examination of the ballots revealed that over a quarter of the respondents did not have a unique first choice. A sincere poll conducted without these respondents elected Powell with 39.5% of the vote. Ballot-by-ballot DSV without these respondents was even more stable than with all respondents, electing Powell 96.9% of the time. This increase in stability highlights the fact that much of ballot-by-ballot DSV's instability is due to indifferent voters. It might be argued that if voters are indifferent — either individually or collectively — then it is appropriate for a voting system to return unstable results. Indeed stable results essentially misrepresent the winner as being more preferred by the electorate when in fact he or she is equally preferred to one or more other candidates.

Trajectory Selection Simulation

One of the most well-documented examples of the use of decision analysis to facilitate real-life group decision-making in the literature is Dyer and Miles' account of the selection of trajectories for NASA's 1977 Mariner Jupiter/Saturn project [42]. About 80 scientists divided into ten science teams were asked to select a single pair of spacecraft trajectories that would provide the best data collection opportunities for all teams. To facilitate this decision the science teams were asked to each provide their ordinal and cardinal preferences for 32 pairs of proposed trajectories (numbered consecutively from 1 to 36 with numbers 6, 12, 14, and 16 deleted). These preferences (shown in Figure 5.2 with the science team names abbreviated at the top of each column) were then aggregated using several collective choice rules — some which used ordinal rankings and some which used cardinal utilities — and the results were presented to the scientists. Depending on the collective choice rule used, pair 26, 29, or 31 appeared to be the best collective choice. (These pairs were ranked in the top three by every collective choice rule considered.) After some discussion the scientists selected pair 26 and asked that it be modified slightly.

We used the science teams' cardinal utilities to simulate the trajectory pair selection under DSV. This simulation was different from the others we performed in that it involved

Pair	RSS	IRIS	ISS	PPS	UVS	CRS	LECP	MAG	PLS	PRA
1	.722	.871	.850	.550	.820	.850	.700	.500	.600	.546
2	.772	.875	.750	.530	.790	.910	.830	.480	.750	.547
3	.600	.925	.960	.520	.910	.840	.450	.350	.900	.109
4	.600	.948	.700	.570	.970	.840	.480	.351	.900	.108
5	.600	.943	.670	.600	.920	.880	.900	.800	**1.0**	.600
7	.600	.750	.600	.500	.810	.800	.430	.360	.600	.107
8	.600	.886	.900	.780	.830	.860	.740	.700	.700	.556
9	.600	.930	.800	.650	.760	.890	.610	.490	.700	.552
10	.887	.935	.720	.720	.800	.960	.600	.400	.700	.558
11	.600	.928	.920	.810	.720	.910	.490	.260	.500	.106
13	.652	.915	.750	.520	.830	.810	.320	.280	.600	.105
15	.887	.869	.800	.580	.840	.870	.650	.430	.750	.543
17	.600	.966	.850	.750	.830	.850	.770	.510	.700	.557
18	.600	.944	.900	.630	.790	.880	.560	.485	.600	.104
19	.600	.873	.900	.520	.760	.880	.500	.300	.600	.103
20	.772	.937	.620	.550	.630	.860	.580	.350	.600	.549
21	.600	.925	.970	.510	.790	.840	.440	.575	.900	.102
22	.600	.869	.900	.580	.640	.910	.630	.405	.600	.542
23	.600	.875	.800	.530	.670	.910	.780	.408	.600	.545
24	.887	.919	.700	.520	**1.0**	.840	.300	.250	.600	.101
25	.600	.947	.670	.530	.840	.970	.960	.950	.750	**1.0**
26	.600	.996	.900	.700	.850	.870	.980	**1.0**	**1.0**	.990
27	.600	**1.0**	.650	.610	.930	.870	**1.0**	.970	.950	.980
28	**1.0**	.876	.700	.640	.830	.850	.720	.485	.750	.554
29	.600	.957	.770	.750	.840	.930	.990	.990	.950	.970
30	.600	.878	**1.0**	.690	.760	.870	.750	.420	.700	.553
31	.600	.987	.940	.720	.860	.900	.975	.980	.950	.960
32	.600	.890	.770	.550	.720	.880	.620	.570	.700	.551
33	.600	.873	.930	.710	.800	.890	.590	.573	.700	.550
34	.600	.921	.770	.810	.830	.860	.670	.600	.750	.548
35	.600	.883	.980	.820	.830	.880	.730	.595	.700	.555
36	.600	.813	.750	**1.0**	.770	.850	.570	.594	.600	.544

Figure 5.2: Mariner science team cardinal utility values

a small number of voters and a large number of alternatives. In this type of voting situation ballot-by-ballot DSV is highly dependent on chance.

In 1000 ballot-by-ballot DSV simulations, pair 27 averaged 23.7% and was selected 40.3% of the time. Pair 26 averaged 34.5% and was selected 20.3% of the time. Pair 25 averaged 22.8% and was selected 23.2% of the time. The only other pair to be selected was pair 5, which was selected less than 1% of the time. Batch DSV with small S^2 values selected pair 26 with 50% of the vote. Due to the small number of voters and large number of alternatives, no strategic voting occurred under batch DSV with large S^2 values.

The top three pairs selected by our simulations (25, 26, and 27) were ranked in the top 9 by every collective choice rule used in Dyer and Miles' study. Pair 26 was ranked in the top 3 by every rule.

Our DSV simulations never selected pairs 29 or 31, which were in the set of pairs selected by the other collective choice rules. This is due to the fact that neither of these pairs was ranked first by any science team. This observation highlights the fact that DSV systems that use the plurality rule will never select an alternative that is widely preferred as a second choice unless it also makes a strong showing as a first choice. This is not a property of DSV systems in general, rather it is a property of plurality rule systems. A DSV system that used an approval voting rule would not have this property. In a case such as this where selecting a good compromise alternative is a high priority, DSV with approval voting might prove more useful than DSV with plurality rule.

Another interesting observation about DSV in this situation is that it does not take into account the fact that some popular alternatives may be strongly disliked by a minority. For example, choice 27 is ranked first by ballot-by-ballot DSV despite being ranked 30^{th} by one of the science teams. In the collective-choice rules that take into account each voter's entire preference order, a low ranking such as this would hurt a popular alternative, acting effectively as a negative vote. As with all single-vote systems, DSV systems do not factor in low rankings. However, unlike plurality-rule elections, in elections where DSV is used, the data is available for analysts to determine whether the selected alternative was strongly disliked by a minority.

5.3 Case Study: The 1992 US Presidential Election

The 1992 United States Presidential election provides a familiar context in which to explore declared-strategy voting. The election results themselves reveal little of the voter preference information needed for this analysis; however, data from the 1992 American National Election Study (NES) conducted by the Center for Political Studies at the University of Michigan includes more detailed voter preference information as well as other relevant information.

The NES data set contains data from 2485 respondents. The respondents were asked to rate each of the main candidates in the US Presidential race on a "feeling thermometer" scale that ranged from 0 to 100 — the higher the number, the more favorable the respondent's feeling toward that candidate. This feeling thermometer data can serve as a proxy for utility values. It is also useful for analyzing the extent to which voters behaved strategically in the 1992 Presidential election.

Of the 2485 respondents, 1658 reported voting for one of the three main candidates in the 1992 US Presidential election. Each of these respondent's most preferred candidate was determined by comparing his or her thermometer ratings for all candidates. Data was missing for three respondents, and 358 respondents reported thermometer values that did not indicate a single most preferred candidate (these respondents gave their top rating to two or more candidates). Thermometer data for the remaining 1297 respondents was compared with the actual election vote reported by each respondent. This comparison indicates that 21.5% of Perot supporters (those respondents whose thermometer values indicated they most preferred Perot; they may or may not have actually voted for Perot) voted insincerely, perhaps because they voted strategically. Only 7.7% of Bush supporters and 5.5% of Clinton supporters voted insincerely.

Brams and Merrill performed a similar analysis, but included data from the 131 respondents who gave their top rating to Perot as well as one or both of the other candidates. When this data is included, the percentage of Perot supporters who voted insincerely jumps to 34.5% [20].

The NES data set also contains information about how long before the election each respondent decided how to vote. Responses are classified into 12 main categories plus 'other', 'don't know', and 'not applicable'. Figure 5.3 presents a graph of the responses from the 1398 respondents whose answers to this question fell into one of the 12 main categories and whose thermometer data indicated a single preferred candidate. Only 11 of the 12 categories appear in this set, and two pairs of categories have been combined. Thus, only nine categories are shown in this graph; the original category numbers remain as part of the axis labels. The figure indicates that Perot supporters were more likely than Clinton and Bush supporters to decide how to vote one month or less prior to the election. While 74% of Bush supporters and 64% of Clinton supporters decided for whom to vote more

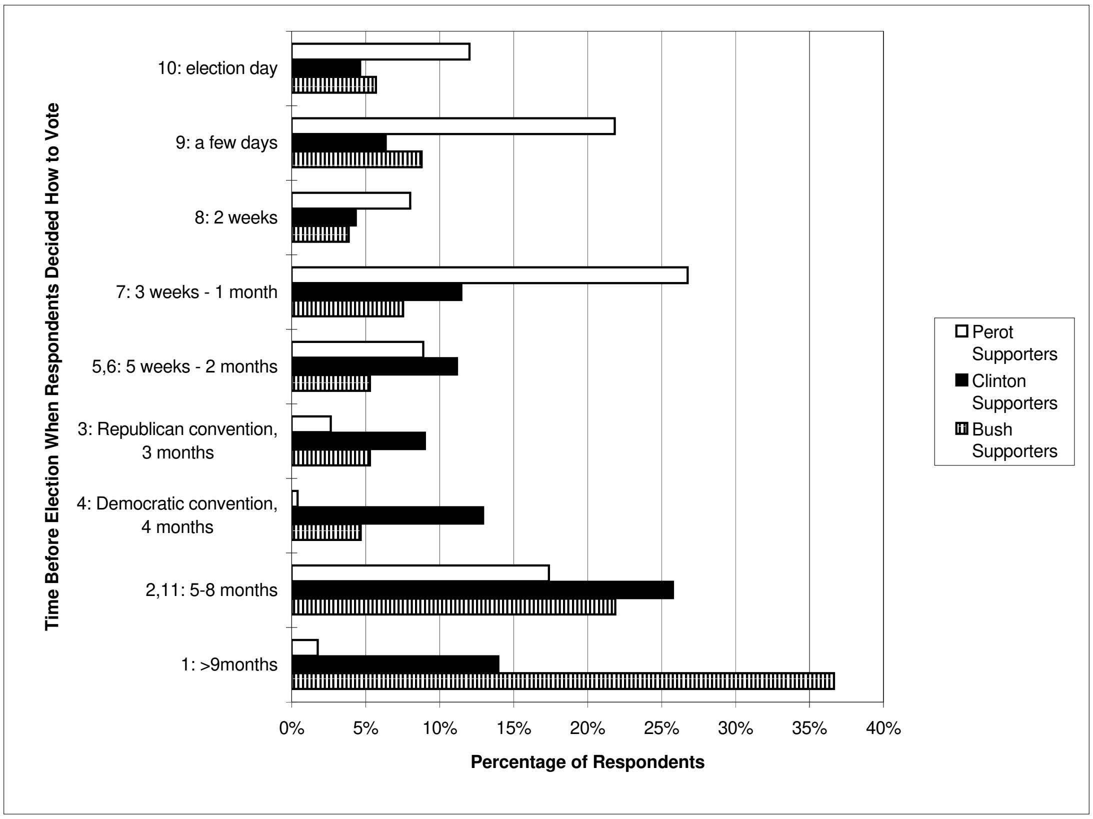

Figure 5.3: Time before election when voters decided how to vote

than one month before the election, only 31% of Perot supporters had made up their minds that early. This may be due in part to the fact that Perot had temporarily dropped out of the Presidential race in July and did not re-enter until a month before the election. However, when we examine the percentage of voters who decided how to vote in the last few days before the election or on the day of the election, we see that 34% of Perot supporters made their decision during that period as compared with 15% of Bush supporters and 11% of Clinton supporters. This may be indicative of strategic behavior among Perot supporters.

Figure 5.4 provides further evidence of strategic behavior. This graph includes only the data from the 258 Perot supporters whose answers to the above question fell into one of the 12 main categories. Fifty percent of the Perot supporters who ended up voting for Bush made their decision within the last few days before the election or on election day. Only 28% of those who ended up voting for Clinton and 23% of those who ended up voting for Perot made their decision during that period. Given that the polls published during the week of the election [51] predicted that Clinton would win with over 42% of the popular

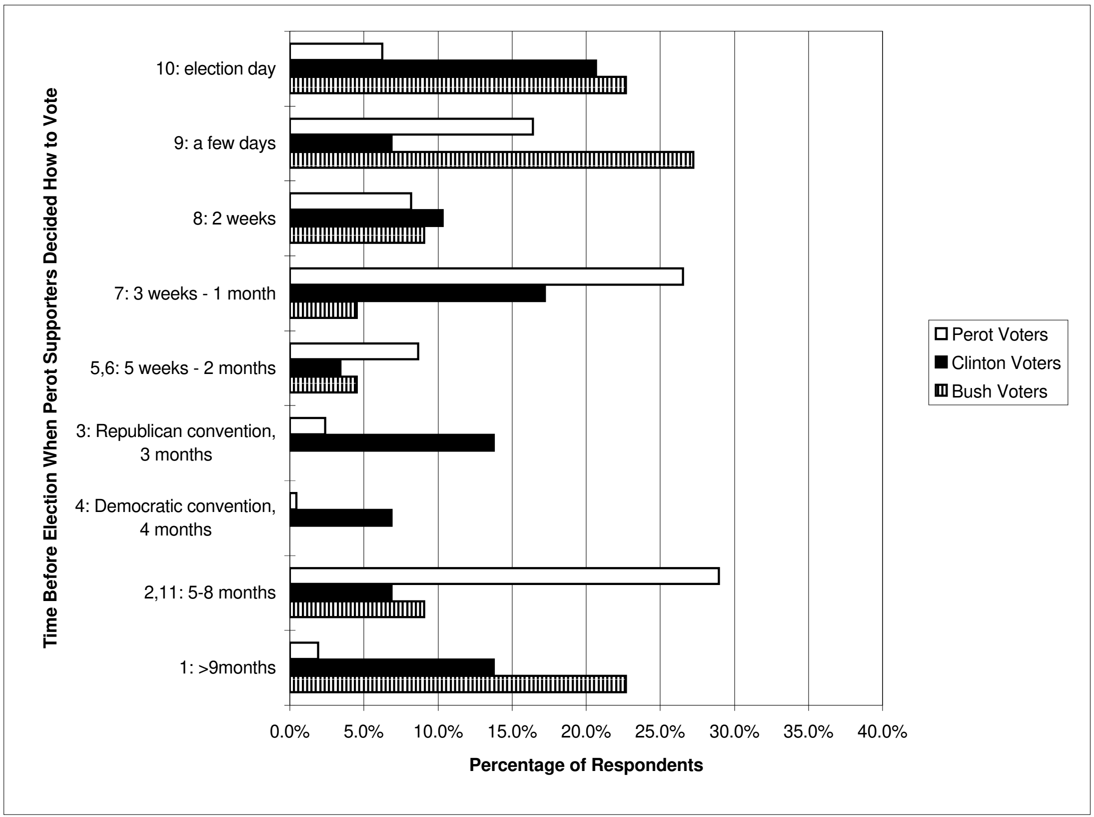

Figure 5.4: Time before election when Perot supporters decided how to vote

vote and Perot would come in third with under 16% of the popular vote, the voters who had the most to gain by voting strategically were the Perot supporters who preferred Bush to Clinton. Had sufficient numbers of these voters voted strategically without Perot supporters who preferred Clinton to Bush doing likewise, they might have secured a victory for their second choice candidate.

It is also useful to examine the results of some of the Gallup Polls conducted prior to the 1992 Presidential election. A survey conducted during 13–15 October (less than 3 weeks before the election) asked voters whether there was any chance they would vote for a candidate other than the one they said they planned to vote for. Seventy-eight percent of Perot supporters said they might vote for another candidate as opposed to 43% of Bush supporters and 37% of Clinton supporters. When Perot supporters were asked what they would do if, in November, it looked like Perot could not win, 36% said they would vote for Perot anyway, while 26% said they would switch to Bush, 30% said they would switch to Clinton, and 8% expressed no opinion [51]. These results also suggest strategic behavior.

We used the NES thermometer data to simulate the 1992 US Presidential election under declared-strategy voting. Although DSV would not have changed the election winner, it would have likely revealed far more information about voter preferences than was revealed through plurality voting. In the actual 1992 election, Bush received 37.7% of the popular vote, Clinton received 43.3% of the popular vote, and Perot received 19.0% of the popular vote. The NES data indicates a slightly higher popular vote percentage for Clinton (post election surveys often show more support for the winner than election results indicate). Our DSV simulations all resulted in a much higher vote count for Clinton and a much lower vote count for Perot.

Our DSV simulations used the NES thermometer data from all 2171 respondents for which it was available, regardless of whether they reported voting. In order to study the effect of indifference we conducted our simulations with the complete thermometer data set and again with data from only the 1820 respondents who indicated a single first-choice candidate. Ballot-by-ballot simulations conducted with the 2171 respondent data set had an outcome stability of 97.1% while those conducted with the 1820 respondent data set had an outcome stability of 99.0%. The results of the actual election, polls, and our simulations are summarized in the table below.

1992 US Presidential Election: actual election results, polls, and simulations

	Bush	Clinton	Perot
Popular vote in actual election	37.7	43.3	19.0
NES self-reported election vote	33.9	47.6	18.1
Plurality vote synthesized from NES data (1820 respondents)	34.3	48.6	17.1
Batched DSV from NES data (1820 respondents), large S^2	34.1	49.0	16.6
Batched DSV from NES data (2171 respondents), large S^2	31.7	54.7	11.7
Batched DSV from NES data (1820 respondents), small S^2	39.6	56.5	3.6
Batched DSV from NES data (2171 respondents), small S^2	36.3	60.7	3.0
Ballot-by-ballot DSV from NES data (1820 respondents)	38.2	56.2	5.3
Ballot-by-ballot DSV from NES data (2171 respondents)	34.6	59.3	6.0

5.4 Evaluation Criteria

The evaluation criteria introduced in Chapter 2 are useful for exploring the properties of declared-strategy voting from a theoretical standpoint. These criteria as applied to DSV with plurality rule are summarized in Figure 5.5. When DSV is used in conjunction with other voting rules it may have slightly different properties.

The Condorcet Criterion

DSV systems inherit some properties from the positional voting procedures they employ. Because none of the single-stage positional systems are guaranteed to select a Condorcet winner, DSV systems do not satisfy the Condorcet criterion. However, as with some positional systems, some DSV systems may guarantee that the Condorcet loser is never selected.

Brams and Fishburn have analyzed the ability of plurality and approval voting to select a Condorcet winner when voters use poll data to vote strategically [17]. They concluded: "Under plurality voting, the Condorcet candidate will always lose if he is not one of the top two candidates identified by the poll" and "the Condorcet candidate will always win if he is one of the top two candidates identified by the poll." They also concluded: "Under approval voting, a non-Condorcet candidate may be helped, and the Condorcet candidate hurt, by a poll, whether or not the Condorcet candidate is one of the top two candidates identified by the poll" and "adjustments caused by continual polling may (i) have no effect on the outcome, (ii) reinforce or undermine a particular outcome, or (iii) induce cycling." This analysis is applicable to DSV systems that employ plurality or approval voting as these systems essentially simulate what happens when voters use poll data to vote strategically. While it is certainly a deficiency of plurality DSV that the Condorcet candidate cannot win unless it is one of the top two in the polls, it is a deficiency shared with plurality voting in many real election situations. A similar problem occurs also in the STV system when the Condorcet candidate is the candidate with the fewest first-place votes.

Consistency, Monotonicity, Neutrality, and Pareto-Optimality

DSV systems have several properties in common with single transferable vote (STV) systems. When DSV is used in conjunction with plurality rule (with each voter casting a single vote, regardless of the number of alternatives) it is indeed quite similar to STV. Both systems allow voters to cast a ballot with ordered choices that is evaluated into a single vote. In fact, STV is essentially a simple DSV system (absent preference intensities) in which second-choice voting occurs only when there is complete certainty that a voter's first choice has enough votes to win or near certainty that a voter's first choice cannot win. As with STV, DSV is Pareto-optimal and neutral, but not necessarily consistent or monotonic. The difficulty of exploiting these violations in either system make the lack of consistency and monotonicity insignificant for most practical purposes.

Voting Procedure	Type	Always Selects C-Winner	Never Selects C-Loser	Consistent	Independent	Monotonic	Neutral	Pareto-Optimal	Manipulability
Batch DSV with plurality-rule	Multi-stage/ Positional	✗	✗	✗	✗	✗	✓	✓	Manipulable with sophisticated strategies only
Ballot-by-ballot DSV with plurality rule	Positional	✗	✗	✗	✗ (also unstable)	✗	✓	✓	Manpulable with sophisticated probabilistic strategies only

Figure 5.5: Properties of declared-strategy voting

Independence from Irrelevant Alternatives

DSV violates the independence from irrelevant alternatives criterion for the same reason that STV violates it: the outcome may be affected by the removal or addition of alternatives. However, this violation becomes more serious in DSV systems that involve randomness. When randomness is involved, the same profile of ordinal preferences will not necessarily result in the same election outcome every time the votes are counted. The Cambridge implementation of STV suffers from the same problem, but is of little practical concern because the effects of randomness have proved to be quite minor and result in little instability [47]. But the instability associated with some types of DSV may render them unacceptable for some purposes.

Tendency to Encourage Manipulation

As with STV, DSV results in elections with high average effectiveness because it allows voters to be effective even when their first choice is not likely to win. This in turn discourages manipulation because manipulation is not necessary to cast an effective vote. Furthermore, the complexity of both systems make opportunities for manipulation difficult (although not impossible) to identify when there is a large number of voters.

When the number of voters is small, opportunities for manipulation may become apparent to knowledgeable voters using batch DSV. For example, suppose 21 voters hold the following ordinal preferences for three alternatives.

Preference orderings	$a\,b\,c$	$a\,c\,b$	$b\,a\,c$	$b\,c\,a$	$c\,a\,b$	$c\,b\,a$
Number of voters	5	5	0	5	5	1

Thus if all voted sincerely in a plurality election, a would win with 10 votes and b would finish last with 5 votes. Assuming the appropriate cardinal preferences (not shown for the sake of simplicity) batch plurality rule DSV would result in the supporters of b all voting for c, their second choice. Thus the results of the DSV election would be as follows:

	a	b	c
First Round	10	5	6
Last Round	10	0	**11**

Now suppose that voters with preference ordering abc are able to obtain the cardinal preference information for all the other voters. If these voters have sufficient knowledge they will figure out that if three of them adopt a sophisticated strategy, misrepresenting their preferences as bac, they can change the outcome of the election. With three voters misrepresenting their preferences, the ordinal preferences appear to be:

Preference orderings	$a\,b\,c$	$a\,c\,b$	$b\,a\,c$	$b\,c\,a$	$c\,a\,b$	$c\,b\,a$
Number of voters	2	5	3	5	5	1

With this preference ordering, the batch plurality DSV results are:

	a	b	c
First Round	7	8	6
Last Round	**12**	9	11

Thus, manipulation has changed the outcome of the election. In this case other voters may adopt counter strategies that will prevent this manipulation from being successful, assuming they posses sufficient knowledge and information. But the only way manipulation can be rendered impossible is if sufficient amounts of randomness are built into the DSV system. In ballot-by-ballot DSV, manipulation is only possible by using sophisticated probabilistic strategies that take into account the probability that the ballots will be calculated in a particular order.

From the knowledge-needed-to-manipulate perspective, DSV without randomness is still more difficult to manipulate than the positional voting systems because knowledge of voters' ordinal preferences is not always sufficient; knowledge of voters' complete cardinal preferences may be required.

Expressiveness

The concept of expressiveness, introduced in Chapter 3, is rarely discussed in evaluations of voting systems. This may be due to the fact that voting systems are generally thought of only as a means to make a decision; thus, they are evaluated only in terms of their ability to make a good decision. But once a decision has been made it is often important that people understand why the decision was made. Especially when the decision is the selection of a representative or elected official, it is equally important to understand how much of a mandate the winner really had. It is also important for the satisfaction of the electorate that voters feel that their vote meant something, even if their preferred candidate did not win.

When there are a small group of voters assembled for a meeting, discussions prior to a vote help people to understand the degree to which each alternative is supported and why. Furthermore, a dynamic process often occurs in which those who realize there is little support for their preferred alternative eventually abandon that alternative and argue for their second choice. Lakeman provides a graphic example of this behavior:

> Rowland Hill …records that, when he was teaching in his father's school, his pupils were asked to elect a committee by standing beside the boy they liked best. This first produced a number of unequal groups, but soon the boys in the largest groups came to the conclusion that not all of them were actually necessary for the election of their favourite and some moved on to help another candidate, while on the other hand the few supporters of an unpopular boy gave him up as hopeless and transferred themselves to the candidate they considered the next best. The final result was that a number of candidates equal to the number required for the committee were each surrounded by the same number of supporters, with only two or three boys left over who were dissatisfied with all those elected. This is an admirable example of the use of the STV [63].

Indeed, STV has been praised for simulating the human behavior that occurs in situations where voter preferences are well known. However, STV falls short in its ability to simulate this behavior completely because it does not take into account the intensity of preferences. It is likely that the school boys who favored one boy only slightly more than another were the first to abandon their favorites, while those who strongly favored one boy held out the longest or refused to support a second choice regardless of the electability of their first choice. DSV is unique among the voting systems presented in its ability to model this behavior.

DSV is more expressive than any of the positional voting systems because it allows voters to express the intensity of their preferences and because it allows voters to cast votes that are simultaneously effective and expressive. Although voting systems involving utilities are often criticized because they are based on the arguable assumption that interpersonal comparisons of cardinal measures are meaningful, they do offer voters a better chance at expressing themselves then do positional voting systems. DSV combines the best of both worlds, allowing voters to express themselves with cardinal preferences while employing an aggregation system that does not rely on interpersonal comparisons of these preferences: a voter's cardinal preferences are used only to determine an optimal voting strategy for that individual, but they are not aggregated into the election tally. However, cardinal preference data is available for analysis after the election. This data may be useful, for example, to distinguish between a winner generally preferred only slightly to the runner-up, and a winner greatly preferred to the runner-up. This distinction, which cannot be made with ordinal preference data alone, may be useful for guiding future policy decisions.

DSV is also superior in its ability to allow voters to express preferences extraneous to their utilities for the candidates, but still relevant to the election. Using DSV, voters who are indifferent between the candidates may select underdog, bandwagon, abstention, or random strategies, or other strategies reflective of their preferences. For example, as suggested in Chapter 4, voters who are indifferent between the candidates running in their party's primary, may have a desire to help make their party appear unified. Thus they would choose a bandwagon strategy. Other indifferent voters might enjoy personal satisfaction from knowing they voted or might be interested in contributing to a high rate of voter participation in their precinct. By choosing a random strategy or an abstention strategy these voters can add to the total number of votes cast without influencing the outcome and without registering a preference. Voters who are indifferent between individual alternatives but have an opinion about the alternatives as a group — for example, all are excellent, or none are acceptable — can also express themselves with DSV.

Chapter 6

Conclusions and Future Work

We have presented the declared-strategy voting paradigm and described two types of voting systems that can be implemented under this paradigm: batch DSV and ballot-by-ballot DSV. We have also discussed the problem of rational-strategy formulation and presented an automated strategy formulation system that allows voters to submit only their cardinal preferences for each candidate in order to have their optimal strategies formulated automatically. Our strategy formulator uses the expected-utility model of voting in conjunction with our novel technique for calculating pivot probabilities. We also expand on the expected-utility model, allowing voters to use tie-breaking rules when they are indifferent between alternatives. We have presented simulations of both types of DSV applied to several real and hypothetical voting situations.

The remainder of this chapter is organized as follows. We begin by examining the impact of DSV on those concerned with policy decisions. We then examine issues relevant to the acceptance or rejection of DSV by various populations. While it would be premature to conclude that DSV will be accepted or rejected by a given population, we offer an analysis of some of the factors that might determine its acceptance. Finally, we describe some future directions for both improving DSV and assessing its impact and acceptance by real populations.

6.1 Impacts

The impacts of a decision-making tool may be felt by many types of stake holders. The tool may affect those who make decisions, those who participate in group decision-making processes, those who must live with the consequences of decisions, and those who interpret decisions. Here we assess the impacts of DSV from the perspective of three interrelated groups, policy-makers, voters, and analysts. The ability of voters (or survey respondents) to express themselves, policy-makers to make appropriate decisions, and analysts to interpret and explain election results will also impact a fourth group: those who must live with the consequences of the decisions.

6.1.1 Policy-Makers

In addition to its use as a voting system, DSV may be applied to group decision-making situations in which a single policy-maker must make a decision and may use a non-binding survey or the results of a previous election as a guide. In such situations, the impact of DSV on policy-makers can be analyzed by considering the accuracy with which DSV discovers voter preference information and the extent to which DSV facilitates arrival at a decision.

A known flaw of nearly every method for obtaining election or poll input concerns the extent to which the voters can, should, and do present insincere preference information. DSV, in particular the ballot-by-ballot method, is the first system that elicits sincere preference data without excessively relying on randomness. As compared with other systems, DSV provides clear advantages in terms of discovering sincere preference information from voters.

DSV also facilitates arrival at a decision by highlighting preference patterns that might be overlooked using other decision-making tools. Although further work would be useful for establishing guidelines policy-makers can use in interpreting DSV results, we believe DSV is useful to policy-makers in its current form. The following are examples of areas in which DSV can aid a policy-maker:

- DSV is useful for gauging strength of preference and level of indifference among voters. It can help distinguish alternatives that are strongly favored by their supporters from those that are only slightly favored by their supporters. For example, in our course times survey simulation, the use of DSV revealed that students who preferred 8:00 am classes preferred them only slightly to 6:00 pm classes, whereas students who preferred 6:00 pm classes strongly preferred them to classes held at any other time.

- DSV is useful for identifying sets of alternatives that act as mutual substitutes. These alternatives are all highly rated by the same group of voters, and these voters are either indifferent between members of this set or only weakly prefer one member of the set to another. For example, in our university lecture series survey simulation, DSV revealed that “science” and “health and medicine” were mutual substitutes, favored by the same group of respondents.

- DSV is useful for determining which alternatives voters would support if they knew their favorite was not a serious contender. This allows decision-makers to collect one set of ballots and use them to simulate a run-off election or survey.

6.1.2 Voters

We have also shown that DSV has properties that make it attractive from the perspective of voters. DSV is sufficiently general that it subsumes all other voter preference expression mechanisms currently in use. While such generality is interesting at a theoretical level, there is a far more compelling reason to claim superiority for DSV with regard to expression of voter preference: DSV is the only system to date that simultaneously allows voters to express sincere preference information while still casting their votes strategically. While it remains true that not every voter will be sufficiently educated to develop the best preference profile, DSV affords each voter identical access to the preferences expressed by others. Moreover, the rationally optimal strategy formulator described in Chapter 4 affords all voters equal ability to vote strategically based on their preference profiles.

The availability of computers that can perform relatively sophisticated computations quickly enables us to implement DSV systems in which the burden of strategy formulation does not fall on the voters. In addition, computers make possible systems in which many rounds of balloting may be conducted quickly and without returning to the voters. As a result, voters are able to express themselves sincerely without diminishing the effectiveness of their votes. Furthermore, although computers are necessary for tallying ballots under DSV, they are not required for ballot collection: voters can register their preferences for each candidate on a paper ballot rather than entering them directly into a computer. Thus, DSV offers the following benefits to voters:

- DSV allows voters to vote strategically (thus maximizing the effectiveness of their votes) regardless of whether they have obtained accurate information about the preferences of the rest of the voters or whether they know how to formulate optimal strategies.

- DSV allows voters to express their sincere preferences (including their preference intensities) without sacrificing the effectiveness of their votes.

- DSV allows voters to express their preferences for all contingencies, allowing multi-round elections to be conducted with a single round of balloting.

One drawback to ballot-by-ballot DSV in its current form is that it is unstable, and thus election results may not be repeatable and may be dependent on chance. Although repeating the election many times and averaging the results may prove satisfactory in some circumstances, our acceptability study suggests that voters may have trouble accepting results based in part on randomness.

6.1.3 Analysts

DSV is also useful to those who interpret election results, including scholars, media analysts, and elected representatives who want to understand the opinions of their constituents. To evaluate the impact of DSV on analysts, we examine the kind of information DSV makes available to the analysts and how such information contributes to the analysis.

To the extent that DSV elicits sincere preference information, it is clear that DSV provides more information than do traditional voting systems, in which analysts cannot distinguish sincere from insincere votes. There is still much to be discovered concerning how such information can be useful in explaining voter behavior or understanding the results of a DSV election. In addition to any final tally reported for a DSV election, we believe that analysis of intermediate results may be crucial to obtain an accurate view of election results. Consider, for example, 1000 simulations of a ballot-by-ballot election. At the end, candidates A and B may each show an average of 50% support, but this could occur under the following disparate situations:

- In each round, candidates A and B received approximately half the voters' support.
- In each round, either candidate A or B received nearly all the support, but half of the rounds elected A and the other half elected B.

Voter behavior is quite different in the above two situations, yet this cannot be seen merely by examining the end-result of DSV computations. Further research is needed to determine how the information harvested from a DSV election can help explain voter and election behavior. It is our belief that while DSV makes analysis more difficult, it will ultimately make analysis more revealing.

6.2 Acceptance

As evidenced by our acceptability study, reported in Chapter 5, acceptance of DSV depends not only on proof (or convincing arguments) that DSV is theoretically better than traditional systems; there are psychological, political, and sociological aspects that must be considered along with objective scrutiny of DSV. We now examine issues relevant to the acceptance of DSV for polling and voting. Our approach is first to examine those conditions that would imply eventual acceptance of DSV, and then to examine the likelihood of those conditions occurring. The discussion here is broadly based on the theoretical and normative properties associated with DSV as well as historical precedent and context that have heralded the acceptance of other voting systems.

Our analysis in Section 2.3 suggests that people are most likely to adopt new voting systems at times of great change or following elections with perverse or undesirable outcomes. In elections for which plurality rule is used, such perversity often occurs when more than two alternatives are under consideration. DSV might be considered as a remedy in such situations. Although governmental elections may raise awareness about the manipulability of traditional voting systems (for example, the presence of third party Presidential candidates tends to promote discussion about whether it is wise to vote for a candidate with little chance of getting elected), DSV is unlikely to be introduced for these elections without first becoming well established as a system for non-governmental elections. Organizations with educated members who must frequently choose among many alternatives are likely to be among the first to adopt DSV.

Our acceptability study suggested that, at least for a well-educated population, voters are open to the idea of a new voting system and the concept of voting strategies. However, it also demonstrated that education about DSV and voting in general will be essential before DSV will gain acceptance. The aspect of DSV that raised the most objections among our survey respondents was instability. Except in situations where a vote is for advisory purposes only or where voters are willing to accept an outcome based on some randomness, the instability of ballot-by-ballot DSV is likely to prevent its acceptance. If stability can be improved or presented in a way that makes it more palatable, or if batch DSV can be improved, the acceptability of DSV is likely to increase substantially. We believe this factor is significantly more important for acceptance than DSV's violations of some of the normative properties discussed in the previous chapter, as the degree to which traditional voting systems possess these properties does not seem to be indicative of those systems' popularity.

6.3 Future Work

Our work introduces a new voting paradigm about which many questions remain unanswered and many variations and extensions remain to be explored. Listed below are several areas that look especially attractive for future exploration.

Acceptability. We conducted a limited acceptability study by surveying a group of well-educated people about one type of DSV. Further work is needed to gauge the acceptability of other types of DSV and the acceptability of DSV among a more general population. In addition, we recommend conducting a study in which a professional organization or other voting body conducts a DSV election shadowing an actual election after its members are educated about DSV and their current voting system.

DSV combined with approval voting or other voting rules. We have limited our study to DSV systems that use the plurality voting rule. Our simulations with many alternatives (especially our trajectory selection simulations) highlighted the fact that the plurality rule prevents the selection of an alternative that does not receive a large number of first-place votes, regardless of the number of second-place votes that alternative receives. DSV systems that use an approval voting rule would not have this problem. The combination of DSV with approval voting, Borda count voting, and other voting rules should be explored.

Tie-breaking rules. We introduced the concept of tie-breaking rules to augment the expected-utility model of voting. However, more work is needed to determine what types of tie-breaking rules are useful in actual voting situations and the impacts of these rules on the behavior of DSV systems.

Uncertainty. As we demonstrated, uncertainty is an important factor in the calculation of pivot probabilities. We introduced a method for accounting for the uncertainty of predictions in ballot-by-ballot DSV and suggested some possible approaches to account for uncertainty in batch DSV. Further work is needed to explore ways of incorporating voters' attitudes towards risk, the entire set of voter preferences, or the maturity of the election in the calculation of uncertainty for batch DSV.

Stability. In many actual voting situations, voters are unlikely to accept an unstable voting system. We have demonstrated that batch DSV is stable, assuming the use of an arbitrary factor for uncertainty and an arbitrary termination condition in case it fails to converge. However ballot-by-ballot DSV is generally unstable. Further work is needed to develop a DSV system that is stable without relying on arbitrary factors. We envision a system that would allow stability to be adjusted depending on the importance of picking a single winner versus preventing manipulation or gaining a greater understanding of the preferences of the electorate.

Dependent alternatives. We have only explored the use of DSV for decisions between independent alternatives. However, decision-makers must often select several dependent alternatives. The use of DSV in decisions involving dependent alternatives should be explored.

Sophisticated strategies. We demonstrated that batch DSV systems may be manipulable by voters who use sophisticated strategies. Further work is needed to determine the extent to which sophisticated strategies are feasible to develop in actual voting situations as well as the possibility of designing a strategy formulator that can formulate optimal sophisticated strategies for voters.

Analysis of election and survey results. One of the positive characteristics of DSV systems is that they provide complete (and likely sincere) cardinal preference information for the entire electorate. This data should prove useful to decision-makers using a DSV survey to guide their decisions, and to analysts interpreting the results of a DSV election. Further work is necessary to determine what methods are most useful for interpreting DSV data.

These areas are a few of the many areas where further work might be focussed. In the process of introducing this new paradigm we have attempted to sketch a rough overview of this previously unexplored concept, filling in as many details as is possible during a first exploration. In trying to map out this paradigm we encountered many unexpected problems. Happily, we also encountered some unexpectedly simple solutions. But as we conclude our work, we leave many problems unsolved, and probably many more undiscovered. It is our hope that other investigators will find some of these problems worthy of their attention.

Appendix A

Survey Instrument

PAGE 1

Survey

This survey is being conducted by Lorrie Cranor as part of a graduate research study. The questions posed are hypothetical. The results of this survey will not be used to influence campus policy. Please answer the survey questions as you would if the results would affect you, even if you will not be a student next semester.

1. Did you attend school full time during the Spring 1996 semester? _____ yes _______ no

2. The university lecture series committee plans to host one well-known speaker each week during the upcoming academic year. For each potential lecture topic, please circle the number that most closely corresponds to your interest level.

	Not at all interested										Extremely interested
Anthropology	0	1	2	3	4	5	6	7	8	9	10
Gender issues	0	1	2	3	4	5	6	7	8	9	10
Health and medicine	0	1	2	3	4	5	6	7	8	9	10
History	0	1	2	3	4	5	6	7	8	9	10
Journalism/Media	0	1	2	3	4	5	6	7	8	9	10
Literature	0	1	2	3	4	5	6	7	8	9	10
Politics	0	1	2	3	4	5	6	7	8	9	10
Race relations	0	1	2	3	4	5	6	7	8	9	10
Science	0	1	2	3	4	5	6	7	8	9	10
Sports	0	1	2	3	4	5	6	7	8	9	10

3. The dean would like to determine the optimal time of day to hold a course you want to take next semester. For each of the possible starting times, please circle the number that most closely corresponds to your interest level.

	Not at all interested										Extremely interested
8:00 am	0	1	2	3	4	5	6	7	8	9	10
10:00 am	0	1	2	3	4	5	6	7	8	9	10
12:00 pm	0	1	2	3	4	5	6	7	8	9	10
2:00 pm	0	1	2	3	4	5	6	7	8	9	10
4:00 pm	0	1	2	3	4	5	6	7	8	9	10
6:00 pm	0	1	2	3	4	5	6	7	8	9	10
8:00 pm	0	1	2	3	4	5	6	7	8	9	10

Please make sure you have circled one number in every row on this page. Then proceed to page 2.

Figure A.1: Survey Instrument Page 1

PAGE 2

4. Did you vote in a student government election during the 1995-96 school year? _____ yes _____ no

5. Are you registered to vote in the US? _____ yes _____ no, not eligible _____ no, eligible but not registered

6. Suppose that each of the candidates listed below is on the ballot for President of the United States. Please circle the number that most closely corresponds to your approval of each candidate.

	Strongly disapprove										Strongly approve
Bill Clinton	0	1	2	3	4	5	6	7	8	9	10
Bob Dole	0	1	2	3	4	5	6	7	8	9	10
Ross Perot	0	1	2	3	4	5	6	7	8	9	10
Colin Powell	0	1	2	3	4	5	6	7	8	9	10
Dan Quayle	0	1	2	3	4	5	6	7	8	9	10

Most elections in the US use a voting system called *plurality rule.* In this system voters mark a single choice on their ballots, and the candidate with the most votes wins. Imagine that instead of using plurality rule, the 1996 US Presidential ballot asks voters to rate each candidate as you did in question 6. The candidate with the highest rating would be a voter's first choice. One way to tally this kind of ballot would be to have a specially programmed computer analyze each ballot and cast a vote that will maximize the influence of that ballot.

7. Suppose that on the ballot described above your 1st choice ends up coming in last, as shown in the table to the right. However, your 2nd choice comes in first. Your 5th choice comes in a close second. Now suppose that when your ballot was counted, the computer cast your vote for your 2nd choice because it was unlikely that your 1st choice could win and your 2nd choice needed your vote to beat your 5th choice. How satisfied would you be with the performance of the computer in this situation?

Candidate	% of Votes
Your **2nd** choice	33
Your **5th** choice	31
Your 4th choice	21
Your 3rd choice	10
Your 1st choice	5

Extremely dissatisfied Extremely satisfied
1 2 3 4 5

8. Suppose that your 1st choice ends up coming in last again, as shown in the table to the right. This time, however, your 5th choice ends up coming in first. Your 2nd choice comes in a close second. Now suppose that when your ballot was counted, the computer cast your vote for your 2nd choice because it was unlikely that your 1st choice could win and your 2nd choice needed your vote to beat your 5th choice. However, it turned out that even with your vote, your 2nd choice could not win. How satisfied would you be with the performance of the computer in this situation?

Candidate	% of Votes
Your **5th** choice	33
Your **2nd** choice	31
Your 4th choice	21
Your 3rd choice	10
Your 1st choice	5

Extremely dissatisfied Extremely satisfied
1 2 3 4 5

The computer described in questions 7 and 8 uses a new type of voting system called *declared-strategy voting.* Here's how it works: After all of the ballots are entered into the computer, the computer randomly selects one ballot to evaluate. The computer will cast a vote for the highest ranked alternative. After a few ballots have been tallied and a statistically significant trend starts to develop, the computer will take the current election tally into account when evaluating subsequent ballots. For example if your first choice is currently ranked first in the tally, the computer will cast your vote for your first choice. However, if your first choice is ranked last, the computer might determine that a vote for your first choice would be a waste of your vote and vote for your second choice instead. Thus, you would have an opportunity to help your second choice win if it is clear that your first choice has little chance of winning.

Figure A.2: Survey Instrument Page 2

PAGE 3

9. How certain would you want to be that your first choice was not going to win before you supported the computer voting for your second choice?

    _____ I would never want the computer to vote for my 2nd choice
    _____ 100% certain
    _____ at least 90% certain
    _____ at least 80% certain
    _____ at least 70% certain
    _____ at least 60% certain
    _____ at least 50% certain
    _____ I would want the computer to vote for my 2nd choice any time my 2nd choice has a better chance of winning than my 1st choice

10. How easy or difficult do you find it to understand declared-strategy voting?

    Extremely difficult — Extremely easy
    1 2 3 4 5

11. How fair is declared-strategy voting (DSV) compared with plurality voting (where you cast a vote for your first choice only)?

DSV is much more fair		DSV is about the same		DSV is much less fair
1	2	3	4	5

12. How would you feel if DSV were adopted for the US Presidential election?

    Extremely dissatisfied — Extremely satisfied
    1 2 3 4 5

13. How would you feel if DSV were adopted for student government elections?

    Extremely dissatisfied — Extremely satisfied
    1 2 3 4 5

14. How would you feel if DSV was used by the university administration to tally student surveys?

    Extremely unsatisfied — Extremely satisfied
    1 2 3 4 5

15. In your opinion, what are the advantages of declared-strategy voting?

16. In your opinion, what are the disadvantages of declared-strategy voting?

Figure A.3: Survey Instrument Page 3

Appendix B

DSV Simulator Source Code

```
/* code for decision function and related functions from dsvsim4.c
 *
 * declared strategy voting simulation program
 * by Lorrie Faith Cranor (lorracks@cs.wustl.edu)
 */

#define square(x) ((x)*(x))

struct ballot_list {
  int *ballot;  /* array of utility values */
  int *indx;    /* indexes for ballot sorted in ascending order */
  int *tindx;   /* location of utility ties */
  struct ballot_list *next;
};

void decide_vote(ptr, tally, last_tally, altvote, N, B, S, pfunc)
     struct ballot_list *ptr;  /* pointer to one voter's ballot */
     int *tally;  /* current election tally (will be incremented) */
     int *last_tally;  /* tally in BBB, tally from last round in Batch */
     int *altvote;  /* number of voters who have voted insincerely so far */
     int N;  /* number of candidates */
     int B;  /* number of voters */
     double S;  /* uncertaintly */
     int pfunc; /* probability function to use */
{
  int i, j, k, t, found;
  int *E_indx, *E_tindx, *t_indx, *t_tindx;
  double *E;  /* array of expected gains */
  double lb, ub;  /* lower bound, upper bound */
  double param1, param2, param3, param4; /* params to prob. calc. function */
  double M, x, P, U;
  double (*func)();  /* probability calculation function */

  /* setup */
  E = (double *)malloc (sizeof (double) * (N+1));
  E_indx = (int *)malloc (sizeof (int) * (N+1));
  E_tindx = (int *)malloc (sizeof (int) * (N+1));
  t_indx = (int *)malloc (sizeof (int) * (N+1));
  t_tindx = (int *)malloc (sizeof (int) * (N+1));
  iindexx(N, last_tally, t_indx);  /* index last_tallies */
  ifindties(N, last_tally, t_indx, t_tindx); /* find ties in last_tallies
                                               * also randomizes tied
                                               * elements in array */
```

```
if (pfunc == 1) func = euDistanc;
else if (pfunc == 2) func = gaussian3;
else if (pfunc == 4) func = gaussian1;
else {
  fprintf (stderr, "Invalid pfunc\n");
  free (E); free (E_indx); free (E_tindx); free (t_indx); free(t_tindx);
  exit (0);
}
found = 0;

/* if 1st choice is in 1st or 2nd place vote for it and don't
 * worry about probabiliy calculations
 */
for (t = ptr->tindx[N]+1; t>0; t--) { /* loop over 1st choice ties */
  for (i = t_tindx[N]+1; i>0; i--) { /* loop over 1st place ties */
    if (ptr->indx[N-t+1]==t_indx[N-i+1]) { /* 1st choice in 1st place? */
      tally[t_indx[N-i+1]]++;
      found = 1; t = 0; i = 0;
    }
  }
}
if (found) {
  free (E); free (E_indx); free (E_tindx); free (t_indx); free (t_tindx);
  return;
}

/* calculate expected gain */
E[1] = 0;  /* we're never going to vote for last choice, so doesn't matter */
for (i=N; i>1; i--) {
  E[i] = 0;
  for (j=N; j>0; j--) {
    if (i==j) continue;
    if (pfunc == 2 || pfunc == 1) {
      lb = 1.0/3.0;
      ub = 0.5;
      param1 = (double)last_tally[ptr->indx[i]]/(double)B;
      param2 = (double)last_tally[ptr->indx[j]]/(double)B;
      for (k=1; k<=N; k++)
        if(i!=k && j!=k)
          param3 = (double)last_tally[ptr->indx[k]]/(double)B;
      param4 = S;
      E[i] += (double)((ptr->ballot[ptr->indx[i]]-ptr->ballot[ptr->indx[j]])
              * qsimp (func, lb, ub, param1, param2, param3, param4));
    }
    else if (pfunc == 4) {
      U = (double)(ptr->ballot[ptr->indx[i]] - ptr->ballot[ptr->indx[j]]);
      if (U != 0) { /* doesn't matter what P is, if E term = 0 */
        M = (double)last_tally[t_indx[N]]/(double)B;
        x = (double)min(last_tally[ptr->indx[i]],last_tally[ptr->indx[j]])/
          (double)B;
        if (M == x)  ub = M;
        else ub = (square(M) - square(x))/(2 * (M - x)); /* intercept */
        P = (S/2.0)*erf((ub-M)/(S*sqrt(2)))+S*.5;
        if (P == 0.0)
           P = (S/fabs(ub-M))*(1/sqrt(2*M_PI)) * exp(-square(ub-M)/(S*S*2));
        E[i] += U * P;
      }
    }
  } /* end j loop */
} /* end i loop */

indexx(N, E, E_indx);  /* index expected gain */
findties(N, E, E_indx, E_tindx);  /* look for ties in expected gain index */
tally[ptr->indx[E_indx[N]]]++;  /* increment tally */

/* did we vote for someone other than first choice */
```

```
  found = 0;
  for (i = 0; i<= ptr->tindx[N]; i++)
    if (ptr->indx[E_indx[N]] == ptr->indx[N-i]) found = 1;
  if (!found) {
    *altvote = *altvote + 1;
  }

  free (E); free (E_indx); free (E_tindx); free (t_indx); free (t_tindx);
}


/* return smaller of a and b */
int min (a,b)
     int a, b;
{
  if (a < b) return (a);
  else return (b);
}


/* creates an index that identifies ties in an indexed list sorted
 * in ascending order
 *
 * The Nth element of the indx array is the index of the first ranked
 * data element.  If the N-1 element is tied with the Nth element,
 * the Nth element of the tindx array will contain a non-zero number
 * corresponding to the number of elements that tie with the Nth
 * element
 *
 * this function also swaps the indecies for the tied elements with
 * random probability
 */
int findties(n,data,indx,tindx)
int n;
double data[];
int indx[], tindx[];
{
  int i,t,r,temp,first;
  first = 0;
  for (i=1; i<=n; i++) tindx[i] = 0;
  for (i=n; i>1; i--) {
    for (t=i-1; t>0; t--) {
      if (data[indx[i]] != data[indx[t]]) t=0;
      else {
        tindx[i]++;
        if (i == n && tindx[i] > first) first = tindx[i];
        r = (int)random()%2;
        if (r == 1) {
          temp=indx[i];
          indx[i]=indx[t];
          indx[t]=temp;
        }
      }
    }
    i = i - tindx[i];
  }
  return (first+1);  /* first-place tie size */
}


/* for integers, same as above but data is an int */
int ifindties(n,data,indx,tindx)
int n, data[], indx[], tindx[];
{
/* ... */
}
```

```
/***** probability calculation functions and auxilaries *****************/

/* simple distance (note we only implemented this for 3 candidates */
double euDistanc (q, x, y, z,S)
     double q, x, y, z,S;
{
  return (1 - (sqrt(square(q - x) + square(q - y) + square(1 - 2.0 * q - z))));
}


/* Hoffman's method (note we only implemented this for 3 candidates */
double gaussian3 (q, x, y, z, S)
     double q, x, y, z, S;
{
  double D;
  D = sqrt(square(q - x) + square(q - y) + square(1 - 2.0 * q - z));
  return (exp (-.5 * square(D)/square(S)));
}


/* 2-D gaussian method */
double gaussian1 (q, x, S, blank1, blank2)
     double q, x, S, blank1, blank2;
{
  double pi = 3.14159;
  return (exp (-.5 * square(q-x)/square(S))/ sqrt(2 * pi));
}


/***** functions from Numerical Recipes in C ****************************/
/*     note floats have been replaced by doubles throughout             */
/**********************************************************************/

/* from Numerical Recipes in C: The Art of Scientific Computing
 * by William H. Press, Brian P. Flannery, Saul A. Teukolsky,
 * and William T. Vetterling.  Cambridge University Press, 1988.
 *
 * The bodies of these functions have been removed and replaced with
 * blank comments.  Please refer to the above-mentioned text for the
 * complete functions.
 */

/* Indexes an array arrin[1..n], i.e. outputs the array indx[1..n] such
 * that arrin[indx[j]] is in ascending order for j=1,2,...,N.  The input
 * quantities n and arrin are not changed.
 */
void indexx(n,arrin,indx)
int n,indx[];
double arrin[];
{
/* ... */
}

/* same as indexx, but we made arrin an array of ints */
void iindexx(n,arrin,indx)
int n,indx[];
int arrin[]; /* changed from double */
{
/* ... */
}

/* Returns the integral of the function func from a to b.  The parameters
 * EPS can be set to the desired fractional accuracy and JMAX so that
 * 2^JMAX-1 is the maximum allowed number of steps.  Integration is performed
```

```
 * by Simpson's rule
 *
 * modified slightly so that function can take 4 parameters
 */
#define EPS 1.0e-5  /* changed from 1.0e-6 */
#define JMAX 20
double qsimp(func,a,b,x,y,z,S)
double a,b,x,y,z,S;
double (*func)();
{
/* ... */
}
#undef EPS
#undef JMAX

/* This routine computes the nth stage of refinement of an extended
 * trapezoidal rule.  func is input as a pointer to the function to be
 * integrated between limits a and b, also input.  When called with n=1,
 * the routine returns the crudest estimate of int(a->b)f(x)dx.
 * Subsequent calls with n=2,3,... (in tht sequential order) will
 * improve the accuracy by adding 2^n-2 additional interior points
 *
 * modified slighly so func can take 4 parameters
 */
#define FUNC(x) ((*func)(x,g,h,i,S))
double trapzd(func,a,b,n,g,h,i,S)
double a,b,g,h,i,S;
double (*func)();       /* ANSI: double (*func)(double); */
int n;
{
/* ... */
}
```

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